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THE TRANSMISSION OF LIGHT IN DIFFUSING MEDIA

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[Communicated by Mr E. T. Hanson.]

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ABSTRACT. The transmission of light by thin films of diffusing material is investigated, a flicker photometer being used for this purpose. The intensity I of the diffuse light transmitted by a thin film in the direction which is normal to the surface is related to the film thickness x by the empirical equation

$$1/I = C [1 + ax + (ax)^2],$$

where a is a constant depending on the opacity of the material of the film to diffuse light and C is approximately constant for all substances. The ability of a film to hide a contrasting background is proportional to the quantity $H = 1/I$, complete hiding being attained for a value of H which is constant for all the materials investigated, namely a variety of white and coloured paints and papers. An accurate method of measuring the hiding power of such films is described.

(1) INTRODUCTION

THE investigation of the light transmission properties of thin layers of diffusing media is of great practical importance in many industries. Materials such as paper and paint are valued, to a great extent, according to their ability to obliterate or reduce the light contrast of some background with which they are placed in contact, and it is therefore desirable to possess some reasonably accurate method of measuring the optical properties of such substances. Some of the methods employed at present are similar to those used in the measurement of the turbidity of suspensions, i.e. they are based on the determination of the thickness of the film of diffusing material which is necessary to produce visual extinction of some light source.

A critical examination of the inherent weakness of such methods has been made by Toy*, who states that they depend on the ability of the eye to distinguish between the intensities of the light diffused by the film and the light transmitted without scattering. The sensitivity of the eye is dependent on its physiological condition, but when this is normal, a light contrast of approximately not less than 2 per cent. of the field brightness may be perceived. Unfortunately, this value of the least perceptible contrast is not constant for all values of the field brightness, but only when the latter lies within a range producing a retinal illumination from 100 to 10,000 photons. For less intense fields the least perceptible contrast increases rapidly with decrease in field brightness. If the field is always of nearly the same intensity at the extinction thickness of the diffusing medium, comparable results may be obtained. For a medium containing suspended particles of low light absorptive power such a condition is realised. When, however, the medium contains suspended particles of high absorptive power the intensity of the diffused light, i.e. the field brightness, may be very low before the extinction thickness is reached. Therefore, owing to its inability to distinguish light contrasts with the same degree of sensitivity at low as at high intensities, the eye will estimate the extinction thickness to be less than its true value. A consequence of the nature of the observation is that a slight deviation from the normal value of the least perceptible contrast leads to a comparatively large error in the estimation of the extinction thickness. If a black and white surface be covered with a film of white paint of gradually increasing thickness, assuming that the white paint is of such a nature as to leave unchanged the brightness of the white part of the contrasting surface, the black surface will become a lighter and lighter grey as the paint is applied, until a thickness is reached when no contrast between the black and white parts of the surface can be distinguished. When this condition is attained the hiding is complete. Photometric measurements of the changing brightness of the paint-covered black surface give a curve of the form shown in Fig. 1†. The brightness of the covered black surface approaches very slowly to that of the white surface as the paint film increases in thickness, especially for values near the extinction thickness which is given, for the normal eye with retinal illumination in the range of 100 to 10,000 photons, by the condition $YZ/ZX = 98/100$ corresponding to a least perceptible contrast of 2 per cent. of the field brightness. Since there is a very slow approach to the maximum brightness in the region of the extinction thickness, any slight deviation in the value of the least perceptible contrast produces a relatively large deviation in the extinction thickness. Measurements of hiding power made by the cryptometer‡ and other instruments of similar design are particularly susceptible to errors arising in the manner discussed above. There is a further objection to those methods in which the criterion of complete hiding is the extinction of a light source such as a glowing filament, since they do not give any direct indication of the opacity of the medium to diffused light, although in many cases it is this quantity which truly indicates the useful properties

* "Report on Turbidity," *Proc. of the 7th Intern. Congress of Photography*, July 1928.

† Bruce, *Bureau of Standards, Tech. Paper No. 306*, p. 177.

‡ Pfund, *Proc. A.S.T.M.* 30 1930, Pt. 2 (878).

of the substance. For example, a thin film of diffusing material such as paint may not produce obliteration when applied to a contrasting background, although being quite opaque to specularly transmitted light. Such an effect is shown in a pronounced manner by paper which is printed on one side when it is observed by reflected light from the other side. The appearance of a contrast is due, not to specular transmission but to the film being so thin, or the contrasting areas so large, that different intensities of diffused light are apparent.

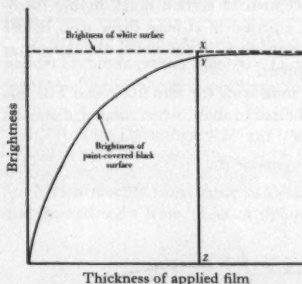


Fig. 1. The effect of covering a black surface with a white film.

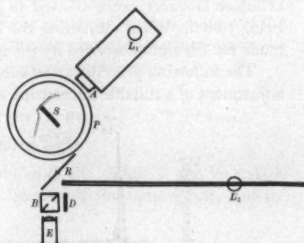


Fig. 2. The flicker photometer.

(2) EXPERIMENTAL METHOD

Apparatus

The flicker photometer illustrated in Fig. 2 was used to measure the specular and diffuse transmissions in non-homogeneous films. The film S is mounted so that it can be adjusted at any angle, indicated on the graduated circle P , to the line of sight through the eyepiece E . Light from a lamp L_1 , rendered parallel by the lens A , illuminates the film at any desired angle of incidence which is again indicated by the graduated circle. A rotating sector, white on the side facing the comparison lamp L_2 and black on the side facing the specimen, is situated at R and is inclined at an angle of 45° to the line of sight through E . The speed of rotation of the sector may be varied, thus allowing the maximum sensitivity to be realised over a wide range of light intensities. By means of a Lummer-Brodhun cube B , and a diffusing screen D , a small flickering field surrounded by a uniform field of approximately the same intensity is obtained, thus ensuring a condition giving maximum accuracy of observation. The lamp L_2 is mounted so that its distance from the rotating sector may be varied at will up to a maximum of 150 cm.

Preparation of the Films

Films were prepared from pigmented media as follows. A thin plate of clear glass was roughly covered with the medium and placed eccentrically on a whirling table which was then rotated at a speed of 1100 r.p.m., thus producing a film of uniform thickness except just near the centre of rotation. A film of uniform thickness having an area of approximately two inches square was obtained in this way, the thickness of the film being controlled by the time of spinning. All films prepared from pigmented media (with the exception of certain quick-drying nitro-cellulose lacquers) were allowed to dry for a period of at least three days before being tested. When measuring the light transmission of the films, allowance was made for the light absorbed by the glass support.

The following procedure was adopted in measuring the film thickness. The fine adjustment of a suitable microscope was calibrated to show, when rotated, distances

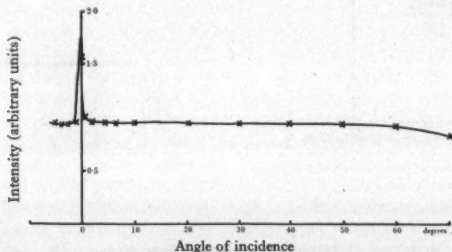


Fig. 3. The intensity of light transmitted by a thin non-isotropic film when illuminated at various angles.

through which the objective travelled. A small section of the paint film was removed, by means of a sharp razor, from the glass support at the spot where the transparency was measured. The microscope was focussed, near the edge of this section, first on the glass surface and then on the top surface of the film. The mean value of several readings at different places round the section was taken and it was possible to distinguish differences of thickness as small as 0.0002 cm. A $\frac{1}{4}$ -inch objective and a 10 $\times$ eyepiece was the most suitable combination for this observation. The thickness of paper used in transparency determinations was measured with a micrometer-screw gauge of the usual form.

Measurement of Specularly and Diffusely Transmitted Light

The intensities of the specular and diffuse transmissions of heterogeneous films were obtained in the following way. The film was illuminated at various angles and the transmitted light emitted normally was measured. Then, allowing for the changing intensity of illumination arising from the variation in the obliquity of the incident

beam, a graph was drawn showing the dependence of the intensity of the transmitted light on the direction of the illuminating beam. From Fig. 3, which is for a thin paint film, it will be seen that the light emitted is of practically constant intensity until the angle of incidence is within one degree of the normal to the surface, when a sharp increase is observed. It appears reasonable to assume from the form of the curve that the intensity of the diffused light for normal incidence is the same as that observed for an angle of incidence of, say, 5° from the normal. Thus the difference of the total intensities at 0° and 5° gives the intensity of the specularly transmitted light.

A decrease in the intensity of the diffused light is noticed when the angle of the incident beam is greater than 50° . This falling off is attributed to the increase in the amount of light reflected specularly for these angles of incidence.

(3) THE DEPENDENCE OF THE TRANSMITTED LIGHT ON FILM THICKNESS

Specularly Transmitted Light

Measurements were made as described above of the diffusely and specularly transmitted light from films of different thicknesses of the same paint, and the

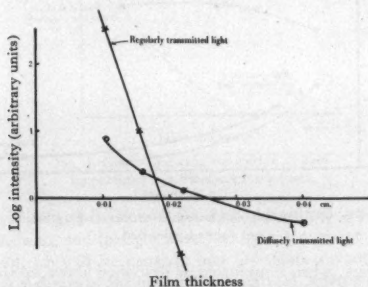


Fig. 4. Transmission curves for a substance of low hiding power.

logarithms of the intensities of the transmitted light were plotted against film thickness. It will be seen (Fig. 4) that the specularly transmitted light decreases with film thickness according to the exponential law*

$$S = S_0 e^{-kx}$$

The intensity of the diffused light decreases much more slowly than the specularly transmitted light as the film thickness increases, so that, for moderate thicknesses, the diffused light becomes the predominating factor in the transmission of the film.

* An exponential equation of this form is derived theoretically in Report No. 2 of the I.E.S. Committee on Glare, *Trans. I.E.S. (N.Y.)*, 10 1915 (362), for the specular transmission in diffusing media.

Experiments were made to investigate more fully the transmission of diffused light by films of various materials, an attempt being made at the same time to correlate this quantity with the ability of the films to hide a surface having regions of definite contrasting brightness. It was thought that the hiding ability might also be a function of the brightness of the surface and therefore measurements were made of this quantity.

Definitions

In the following, the surface brightness and the intensity of the diffusely transmitted light are expressed in terms of the brightness of a standard surface, namely, a piece of finely ground opal glass. The brightness of the standard is defined as the intensity of the light diffusely reflected in a direction normal to the surface when illuminated by a parallel beam of light incident at an angle of 45° . The brightness of the standard opal was 0.69 times that of a magnesium oxide surface. The brightness of a surface is defined as the ratio of the light diffusely reflected in a direction

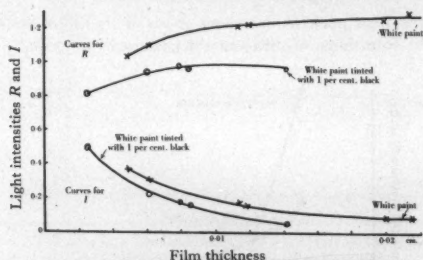


Fig. 5. Reflection and transmission curves for a white and a tinted paint film.

normal to the surface, when illuminated as described above, to the corresponding quantity for the standard opal; the diffusely transmitted light is defined as the intensity of the diffused light transmitted normally by the film, when illuminated at an angle of 45° by the parallel beam, expressed in terms of the brightness of the standard opal when illuminated by the same beam.

The brightness R of a surface, as well as the diffuse transmission I , is a function of the film thickness x , and the brightness of a film of infinite thickness (effectively) will be termed the absolute brightness B .

Diffusely transmitted Light

In Fig. 5 curves are shown for a white paint indicating how the quantities R and I vary as the film thickness increases. The curves are very nearly symmetrically placed about a line parallel to the abscissa, the curve for I being below that for R , this feature being characteristic of all the white materials so far investigated, namely

paints (oil and water media), nitrocellulose lacquers, and papers. Two other important facts are illustrated by these curves: (a) the brightness is approximately constant after a certain thickness is reached and is very nearly equal to the quantity defined as absolute brightness; and (b) the rate of decrease of the intensity of the diffusely transmitted light becomes very slow at moderate film thicknesses, and the actual value of the transmitted light at these thicknesses by no means approximates to zero intensity.

The curves with points marked by circles in Fig. 5 were obtained when the white paint was tinted with a small amount of black pigment. The addition of black pigment displaces both the R and I curves towards the abscissa and modifies the symmetry which characterises the white materials. If we consider a diffusing film of such a thickness that a negligible amount of light is transmitted regularly, an incident beam will be divided into the following five quantities: (a) light reflected

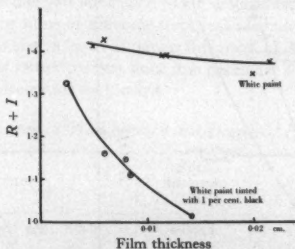


Fig. 6. Curves showing absorption of light by particles in a transparent medium.

specularly, (b) light reflected diffusely, (c) light transmitted diffusely, (d) light absorbed by the medium, and (e) light absorbed by the pigment particles. For any given type of film, (a) will be constant and (d) small compared with the other quantities, so that the sum of the quantities (b) and (c), when a number of films of different thicknesses are considered, must give an indication of the variation of (e) with film thickness. Fig. 6 shows how the quantity $R + I$, which is representative of (a) + (b), varies with film thickness. For white paints $R + I$ is practically constant, indicating that very little light is absorbed by the white pigment particles, while for the white paint tinted with black there is a pronounced decrease in $R + I$ as the film thickness increases, indicating the strong absorptive action of the black pigment particles. This particular point was not investigated further, but development along these lines should lead to valuable information respecting the light absorptive powers of various pigments in liquid media.

(4) DEGREE OF HIDING

The degree to which a film of diffusing material is able to hide a black and white contrasting surface has been compared with its transmitting power for diffuse light. The quantities R/I and $1/I$ were measured for a large number of films of paint of widely differing colours and hiding power characteristics. The films were then sorted into groups containing those producing the same degree of hiding, by comparing visually the abilities of the films to hide a chosen contrasting background with which they were placed in contact. The contrasting background consisted of

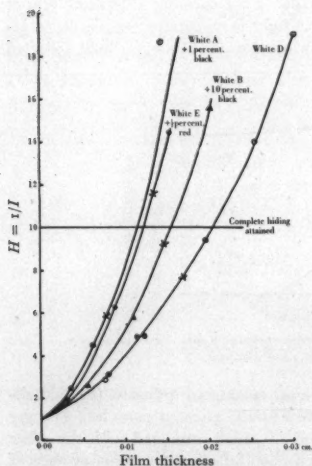


Fig. 7. Transmission curves for white and tinted paints.

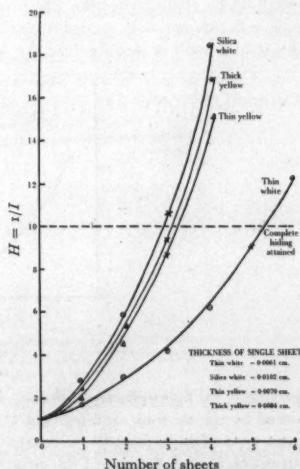


Fig. 8. Transmission curves for white and coloured papers.

two pieces, one of white opal and the other of black glass, their edges being placed in contact. The brightness of the white surface was 83.9 per cent. and of the black surface 0.23 per cent. of that of a magnesium oxide surface. The accompanying table indicates the constitution of some of the films, their thicknesses, and the corresponding quantities R/I and $1/I$. No relation is apparent between the quantity R/I and the degree of hiding, but the latter depends on the quantity $1/I$ in such a way that complete hiding is attained when $1/I = 10$ approximately. This conclusion was found to be further justified by experimental results, too numerous to reproduce, which were obtained for other diffusing films of paint and paper. It

is obvious that the degree of hiding produced by a diffusing screen must be considered in relation to the intensity of illumination and the amount of contrast existing in the background. The background used in these experiments had a contrast as great as any usually met with in practice and the illumination (produced by a 100-watt lamp at a distance of 10 inches from the surface) was fairly intense. For these conditions, the criterion of complete hiding is given approximately by $1/I = 10$. When a magnesium oxide surface is used as the standard for measuring D this condition becomes $1/I = 10/0.69 = 14.49$. Since hiding power is really a property of the diffusing film and not of the background, variation in the latter may be eliminated, for the purposes of definition, by assuming all comparisons of hiding power to be made on a hypothetical background for which the criterion of complete hiding is given accurately by $1/I = 10$. Complete hiding being defined by this quantity, a determination of the film thickness for which $1/I = 10$ will yield a value of the hiding power of the film substance. This determination is easily made by plotting values of $1/I$ for films of different thickness as shown in Fig. 7. For substances available in thin sheets, such as paper, this method is very satisfactory, but for paints the process is rather tedious since it is necessary to prepare at least four films of different thicknesses for each paint.

Comparison of Transparency and Degree of Hiding

Paint film*	Film thickness (cm.)	R/I	$1/I$
White A + 5 per cent. black	0.0033	2.5	3.6 (a)
" + 2 " "	0.0042	3.2	3.8
" + 1 " "	0.0090	3.6	3.8
White B + 5 " "	0.0141	0.6	4.1
White C	0.0121	4.3	4.3
White A + 5 per cent. black	0.0046	3.9	5.1 (a)
" + 1 " "	0.0079	5.7	5.9
" + 1/2 " "	0.0056	5.5	5.6
" "	0.0114	6.9	5.7
White B + 10 per cent. black	0.0108	4.8	5.9
White C	0.0166	6.2	5.8
White A + 5 per cent. black	0.0076	9.1	12.8 (b)
" + 1/2 " "	0.0142	13.0	12.1 (b)
" "	0.0200	14.5	11.6 (b)
White B + 10 per cent. black	0.0146	0.7	9.2 (c)
White D	0.0196	11.0	9.5 (d)

(a) All the films in this group produce approximately the same degree of hiding.

(b) Complete hiding is produced by these films.

(c) The film does not hide completely.

(d) The film produces nearly complete hiding of the contrasting surface.

* The paints, White A, B, C, etc., are made with different white pigments.

(5) EMPIRICAL EQUATION CONNECTING $1/I$ AND x

The time required for a determination of the hiding thickness of a paint may be reduced if a simple relation exists between $1/I$ and film thickness x^* . The form of the curves suggested a parabolic relation and the equation

$$1/I = 1 + ax + (ax)^2 \quad \dots\dots(1)$$

was found to represent the results most satisfactorily and at the same time had the advantage of containing only one unknown constant depending on the nature of the film substance. If $1/I$ is measured in a different unit from that provided by the standard opal glass used in this investigation the equation has the modified form

$$1/I = C [1 + ax + (ax)^2],$$

where C is constant (approximately) for all films and must be determined by a preliminary experiment. The constant a , which may be called the absorption factor for diffuse light, increases with the opacity of the film substance to diffuse light. Fig. 7 shows the accuracy with which this equation represents the experimental results. The curves are calculated from the equation, using one of the experimental points (marked with a cross) to obtain the constant a . The other points on the graph were obtained experimentally and they all fall on or very near to the curves obtained by calculation. Fig. 8 is a similar graph for several different kinds of white and coloured papers. The paints and papers possess widely differing characteristics and the results given in Figs. 7 and 8 are representative of all the materials so far tested throughout the range of film thickness indicated. The slight deviations which do occur appear only for very thin or very thick films, showing that calculation should not be made from films having values of $1/I$ outside the range 4 to 16. If this condition is satisfied, it follows that the hiding power of any material may be obtained from a single observation of $1/I$ on a film of known thickness. Solving equation (1) for x , we have

$$x = [\sqrt{(4H - 3)} - 1]/2a,$$

where $H = 1/I$ and is a measure of the hiding ability of the film. The thickness X for which complete obliteration of the contrasting background is attained corresponds with the value $H = 10$. Therefore $X = 5.08/2a$. Now a is obtained from an observation of H and x for a single film,

$$\text{i.e.} \quad a = [\sqrt{(4H - 3)} - 1]/2x.$$

Therefore the hiding thickness

$$X = 5.08x/[\sqrt{(4H - 3)} - 1] \quad \dots\dots(2).$$

Since H is obtained from the photometer reading, equation (2) allows the photometer to be calibrated so as to give directly the quantity X/x , i.e. the number of films of thickness x required to produce complete hiding.

* Exponential equations for diffusely reflected and transmitted light are derived from theoretical considerations in Report No. 2 of the I.E.S. Committee on Glare (*loc. cit.*) and by Ryde and Cooper, "The Scattering of Light by Turbid Media, Pts. I and II," *Proc. Roy. Soc. A*, 131 1931 (451-75), but are not sufficiently simple for general use.

Three points are worthy of notice in equation (1). Firstly it is seen that, when $x = 0$, $H = 1$. This condition is never realised in practice, but it indicates that for very thin films the transmitted light is approximately equal to the diffused light reflected from the standard opal which is again approximately equal for white films to the light reflected by the film itself. Considerable deviation from this equation would therefore be expected for very thin coloured films but it is found impossible to produce them so thin as to show this. Secondly, when the film thickness is equal to twice the hiding thickness given by $H = 10$, we find $H = 32$, i.e. the transmitted light is $1/32$ of the light reflected from the standard opal which is again of the same order of brightness as the film. When light passes through a film and is reflected back through the film from a background, the effect of the latter should be noticeable until the retransmitted light is reduced to 2 per cent. of the diffusely reflected light. Approximately the same conditions are realised if we allow light to pass through twice the hiding thickness and compare the resulting intensity with that of the diffusely reflected light. From above, the intensity of the transmitted light is about 3 per cent. of the reflected light for twice the hiding thickness, assuming that complete hiding is given by the condition $H = 10$. This of course does not allow for any deviation from perfect reflectance of the background. Lastly, if H_1 and H_2 are the transmission factors corresponding to film thicknesses x_1 and x_2 of any material, then it follows from equation (1) that

$$x_1/x_2 = [\sqrt{(4H_1 - 3)} - 1]/[\sqrt{(4H_2 - 3)} - 1].$$

The right-hand side of this equation contains only H_1 and H_2 , and if these are fixed, for any film we have

$$x_1/x_2 = \text{constant.}$$

In other words, to produce a transmission change from H_1 to H_2 , the film thickness must be changed by a definite fraction of the thickness corresponding to the transmission H_1 , and this fraction is independent of the nature of the film.

(6) THE ADVANTAGES OF THE METHOD

In concluding it is appropriate to enumerate the advantages peculiar to this method of measuring hiding power. It eliminates the possibility of errors arising from the direct use of contrasting backgrounds, and increased sensitivity is attained, since the light is observed after passing only once through the film. The field brightness, when observing at the critical thickness, is always the same and by using an illuminating beam of suitable intensity may be adjusted to lie in the range of brightness giving maximum accuracy of observation. The method is found to be applicable to white and coloured paints with either oil or water media, to white and coloured papers, and is probably applicable to other diffusing materials of a similar nature. There is no reason why the method should not be applied to liquid media containing suspended solid particles.

ACKNOWLEDGMENT

The author wishes to thank the Director and Council of the Research Association of British Paint, Colour and Varnish Manufacturers for permission to publish this paper, and Dr G. F. New for his interest in the work.

DISCUSSION

Mr T. Smith: It is a pleasure to welcome the first paper we have received from the Paint Research Station and many of us hope we shall have the opportunity of discussing other papers on the very interesting group of problems that are being investigated there. Mr Hanstock is to be congratulated on the convenient empirical solution he has reached for this special group of materials. What is essentially the same problem is of importance in other industries, but it does not follow, in the absence of fortuitous common restrictions, that Mr Hanstock's solution is applicable to other materials. While it is easy to see that with certain materials the transmission factor is likely to have a good deal to do with hiding power, we can imagine materials of another kind in which this factor must be of less importance. As far as I know, no theoretical treatment of the general problem has appeared, and I think it would be better for me to offer a theoretical examination as a separate contribution rather than introduce into this discussion considerations which are somewhat foreign to the nature of the paper.

Mr J. Guild: This paper is an example of the systematic effort which the staff of the Paint Research Station are making to place the many and varied problems of this industry on an organised scientific basis. Some of us are personally familiar with the extent and variety of this work and the high measure of achievement attained in the comparatively few years since the Laboratories were established. With regard to the present problem, Mr Hanstock is to be congratulated on the employment of a flicker photometer for his measurements. It is the only instrument fitted for such work, though other workers in similar fields have carefully avoided its use, employing various unsatisfactory makeshifts instead.

It should be pointed out, however, that the flicker photometer, despite its pre-eminent convenience, is purely an empirical instrument and only gives a true comparison of brightness under specified conditions. The flicker field should subtend an angle of 2 degrees as seen by the observer, and the measurements should be carried out at a constant field brightness corresponding to an illumination of 25 metre-candles on the magnesium oxide surface of the flicker disc. Unless these conditions are observed, serious systematic errors may arise in comparisons involving large colour differences.

Dr G. F. New (communicated): Mr Hanstock's paper presents the results of a portion of a general study of the optical properties of paints, embracing reflective, absorptive, and transmissive characteristics of white and coloured media. I congratulate Mr Hanstock on the economy of procedure with which he has analysed the problem and on the simplicity to which he has reduced its general expression.

One of the most interesting features of the work is the promise which it gives of elucidating the dependence of absorption and transmission on the state of dispersion of a mainly absorptive or coloured pigment in a surround of mainly scattering or white pigment. A thin layer of such mixed pigments may vary in hue, saturation, and brightness to a large extent as the state of mechanical dispersion of the pigments

is changed, and the method here developed should be of great service in following such changes.

Mr Hanstock: I should like to thank Mr Guild for his criticism of the flicker photometer described in the paper. In this apparatus the condition for the angular dimensions of the flicker field is satisfied, as is also that for the illumination of the sector when films are being tested which have thicknesses approximating to the value giving complete hiding. At the same time it is realised that it would be an improvement to vary the intensity of light illuminating the film rather than that illuminating the sector, although with this arrangement control of the type of incident illumination might not be so simple.

The empirical equation given in the paper is not claimed to represent the experimental results with such accuracy as might be attained by using an equation of more complicated form derived either theoretically or empirically. The sole claim made for the equation is that it is the simplest form which will represent the experimental results over a restricted range with sufficient accuracy to allow its use in estimating the hiding thickness from observation on films of thicknesses near the value producing complete hiding. Films of this type are the only kind which are of interest to manufacturers of paint and allied materials.

THE HIDING POWER OF DIFFUSING MEDIA

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ABSTRACT. From theoretical considerations an expression is constructed as a numerical measure of the power of a sheet of a diffusing medium to hide the brightness contrasts of a surface on which it is laid. The dependence of this factor on the transmission and reflection factors of the sheet is exhibited and the effect of varying the thickness of the sheet is discussed. A comparison is made of these theoretical results with published experimental observations. The definition adopted for the hiding power is $50/(\text{residual percentage contrast})$. The properties of all sheets may be expressed in terms of two constants, of which one is the reflection factor for an infinitely thick sheet, and the other is a factor for converting sheet thicknesses to the proper numerical scale. For a non-absorbing medium the absolute hiding power is $1 + 2xt$, and the reflection factor $xt/(1 + xt)$, where t is the thickness of the sheet. For an absorbing medium the absolute hiding power is

$$1 + \lambda \sinh 2kt + \lambda (\kappa - \lambda) (\cosh 2kt - 1)$$

and the reflection factor $\lambda/(\kappa + \coth kt)$ where $\kappa = \cosh 2\epsilon$, $\lambda = \sinh 2\epsilon$, and the reflection factor for a very thick sheet is $\tanh \epsilon$. Expressions are given for the hiding power with respect to any assigned ground contrast.

In experimental determinations the measurement of diffuse reflection factors in a photometric sphere is suggested.

The general theory

A PAPER read recently before this Society by Mr R. F. Hanstock* of the Paint Research Station, Teddington, describes an experimental investigation into the capacity of paints to obscure the light contrasts of the background on which they are laid. Among the results obtained is an empirical expression connecting the hiding power with the thickness of the paint film. The problem, which is of great importance in the paint industry, is closely akin to problems which arise in other industries also. An example of wide interest is the visibility of printers' ink through the page of a book or newspaper.

The principles on which a theoretical examination of this problem can be made have been described in some detail in a paper recently read before the Physical Society†. It happens that the illustrations included in that paper relate to another branch of photometry, but the same general theory is applicable. For a detailed exposition reference to that paper should be made. It will be sufficient here to derive the results to be employed from considerations of a very general character.

* R. F. Hanstock, "The transmission of light in diffusing media," *Trans. Opt. Soc.* **33** 1931-32 (137).

† T. Smith, "The optics of photometric measurements," *Proc. Phys. Soc.* **44** 1931-32 (314).

Let one side of a uniform sheet of matter be illuminated by light of intensity I derived from a fixed source F . Part of this light will be transmitted, part absorbed, and part reflected. Let I_1' be the portion transmitted, and J_1 the portion reflected, which fall upon fixed receivers K and L respectively. If we now increase the illumination I k -fold, I_1' and J_1 will also be increased k -fold. Thus I_1' and J_1 are simply multiples of I . In a similar way if the other side of the sheet is illuminated by light of intensity J' from another fixed source G , and the portions of light reflected and transmitted which fall upon K and L respectively are represented by I_2' and J_2 respectively, I_2' and J_2 are simple multiples of J' . Now if the one side receives the light I at the same time that the other receives the light J' , the light emerging from the two sides is the sum of that emerging from the same sides in the two previous experiments. In other words the total light received by K is I' and that received by L is J where

$$J = J_1 + J_2, \quad I' = I_1' + I_2'.$$

We therefore have homogeneous equations of the first order connecting any three of the four quantities I, J, I', J' . We take as the fundamental relations the two equations

$$\begin{aligned} J' &= AJ - CI \\ I' &= BJ + DI \end{aligned} \quad \dots\dots(1)$$

connecting the light on the one side with that on the other side of the sheet, where A, B, C, D are constants depending on the properties of the sheet and, in the most general case, on the wave-length of the light used. When the materials are not selectively absorbent the coefficients may be regarded as constants independent of the wave-length. By solving equations (1) for I and J we obtain

$$\begin{aligned} (BC + AD)I &= AI' - BJ' \\ (BC + AD)J &= CI' + DJ' \end{aligned} \quad \dots\dots(2).$$

To determine the significance of the constants, in equations (1) put $I = 0$, so that all the incident light falls on one side of the specimen from the source G , and is represented by J' . The light denoted by J is then wholly transmitted, and that denoted by I' wholly reflected. The transmission and reflection factors for the given source and receivers are therefore respectively A^{-1} and BA^{-1} . Similarly from equations (2) by putting $J' = 0$ we obtain transmission and reflection factors $(BC + AD)A^{-1}$ and CA^{-1} respectively. If the specimen is unsymmetrical the two reflection factors may differ, so that B and C need not be equal; they will however necessarily be equal if the specimen is symmetrical and the two sources and receivers are similar and similarly arranged. The ratio of the two transmission factors is $BC + AD$, so that the condition for the equality of these factors is

$$BC + AD = 1 \quad \dots\dots(3).$$

It is obvious that if the paths from F to K and G to L are similar this condition will be satisfied. It is also known that the condition holds for any particular path traversed in opposite directions, whether these paths are symmetrical or not. From these facts and the known properties of dispersive media we may justify the adoption of equation (3) in dealing with the present problem.

With a specimen of the type we are considering the transmission factor is found to be independent of the initial direction of incidence: it is therefore the same as for diffuse illumination. Taking into consideration the law of invariance of transmission factors on reversing a path, it follows that with any particular location of source and receiver we obtain results representative of total transmission with diffuse illumination, and from the symmetry of this condition it is clear that condition (3) holds. We are therefore justified in adopting this condition for the purpose of this discussion.

Equations (1), (2), and (3) may now be written in the more concise form

$$(J' - I') = (J - I) M \quad \dots\dots(1 a),$$

$$(J - I) = (J' - I') \bar{M} \quad \dots\dots(2 a),$$

$$|M| = |\bar{M}| = 1 \quad \dots\dots(3 a),$$

$$\text{where} \quad M = \begin{pmatrix} A & -B \\ C & D \end{pmatrix}, \quad \bar{M} = \begin{pmatrix} D & B \\ -C & A \end{pmatrix} \quad \dots\dots(4),$$

corresponding to the forms used in the investigation already mentioned. The importance of the equations in this form is that they show that when a number of specimens are placed in series the matrix of the coefficients of the compound system is the product of the matrices for the components.

Now suppose that the sheet is laid on a material of diffuse (total) reflecting factor P . The sheet is in effect illuminated by the source F on the one side and by the fraction P of the transmitted light on the other. The conditions to be imposed are therefore

$$J' = PI'$$

or

$$AJ - CI = P(BJ + DI)$$

or

$$\frac{J}{I} = \frac{C + PD}{A - PB}.$$

If two portions of the sheet are backed by material of reflection factors P_1 and P_2 we have

$$\begin{aligned} \frac{J_1 + J_2}{J_1 - J_2} &= \frac{(C + P_1 D)(A - P_2 B) + (C + P_2 D)(A - P_1 B)}{(C + P_1 D)(A - P_2 B) - (C + P_2 D)(A - P_1 B)} \\ &= \frac{2AC + (P_1 + P_2)(AD - BC) - 2P_1 P_2 BD}{P_1 - P_2} \end{aligned}$$

by (3). From Weber's law we should expect the visibility of the differences in the background to be measured by $(J_1 - J_2)/(J_1 + J_2)$, so that the inverse of this quantity may properly be regarded as a measure of the hiding power of the sheet. Let us denote it by h , so that

$$(P_1 - P_2)h = 2AC + (P_1 + P_2)(AD - BC) - 2P_1 P_2 BD \quad \dots\dots(5).$$

Under suitable conditions of illumination the difference in the background would probably be perceptible for values of h less than 20, corresponding to a Weber fraction of 2.5 per cent. Ordinary practical requirements might be met by a somewhat lower value.

Non-absorbing sheets

Equation (5) shows that the hiding power as we have defined it is a somewhat complex function of the joint properties of the sheet and the background. The quantities A , B , C , D may be expressed in terms of measurable properties of the sheet. We have already seen that its transmission factor T is equal to A^{-1} , and its reflection factor R is equal to BA^{-1} and to CA^{-1} , supposing the sheet to be symmetrical. In terms of R and T we then have, with the aid of (3),

$$(P_1 - P_2)h = P_1 + P_2 - 2P_1P_2R + \frac{2R}{T^2}(1 - P_1R)(1 - P_2R).$$

As an extreme case we may take $P_1 = 1$, $P_2 = 0$; the corresponding value of h , which we will denote by H , may be called the absolute hiding power. Thus

$$H = 1 + \frac{2R}{T^2}(1 - R) \quad \dots\dots(6).$$

If now the sheet absorbs no light, $R + T = 1$, and

$$H = 1 + \frac{2R}{T} = 1 + 2X \quad \text{say} \quad \dots\dots(7).$$

More generally in the absence of absorption the properties of a specimen are given in terms of X by the equation

$$M = \begin{pmatrix} 1 + X & -X \\ X & 1 - X \end{pmatrix} \quad \dots\dots(8)$$

so that (7) is a special case of the general equation for non-absorbers

$$(P_1 - P_2)h = P_1 + P_2 + 2X(1 - P_1P_2) + 2X^2(1 - P_1)(1 - P_2) \quad \dots\dots(9).$$

Now when matrices of the form (8) are multiplied the result is equivalent to simply adding their X 's. Since we may regard any sheet as a succession of similar films it follows that we may put

$$X = 2\xi + tx,$$

where ξ represents the ratio of reflected to transmitted light for a single surface of the sheet, t is the thickness of the sheet, and x , like ξ , is a function only of the material of which the sheet is made. The value of ξ is generally small, but a large value of X is necessary to secure a large value of h . In films of good hiding power X may therefore be considered proportional to t .

Absorbing sheets

When any light is absorbed in passing through the sheet the way in which the coefficients of the matrix M vary with the thickness is less simple. The body of the sheet may be considered as composed of n similar thin films, the matrix for each being μ , where

$$\mu = \begin{pmatrix} \alpha & -\beta \\ \gamma & \delta \end{pmatrix}.$$

The matrix for the n films in series is then μ^n . Now μ satisfies the equation

$$\begin{vmatrix} \alpha - \mu & -\beta \\ \gamma & \delta - \mu \end{vmatrix} = 0$$

or

$$\mu^2 - 2\mu \cosh \theta + 1 = 0 \quad \dots\dots(10),$$

where $\alpha + \delta = 2 \cosh \theta$. We adopt this form since it is easy to see from the alternative form for μ

$$\mu = \begin{pmatrix} \frac{1}{\tau} & -\frac{\rho}{\tau} \\ \frac{\rho}{\tau} & \tau - \frac{\rho^2}{\tau} \end{pmatrix}$$

that

$$\begin{aligned} \alpha + \delta &= 2 + \{(1 - \tau)^2 - \rho^2\} \tau^{-1} \\ &= 2 + (1 - \tau - \rho)(1 - \tau + \rho) \tau^{-1}, \end{aligned}$$

and as all the factors on the right are necessarily positive $\alpha + \delta$ cannot be less than 2. On multiplying by μ^{n-1} we get the equation

$$\mu^{n+1} - 2\mu^n \cosh \theta + \mu^{n-1} = 0$$

from which μ^n can be found. The solution is easily developed in the form

$$\mu^n \sinh \theta = \mu \sinh n\theta - \sinh(n-1)\theta \quad \dots\dots(11).$$

This is verifiable by inspection in virtue of the identity

$$\sinh(n+1)\theta + \sinh(n-1)\theta = 2 \sinh n\theta \cosh \theta,$$

on noting that the values given by (11) in the special cases $n=0$ and $n=1$ are correct. As the number of films of a given thickness in the sheet is proportional to the thickness t of the sheet, we may put (11) into the alternative form

$$M = \mu^n = \nu \sinh kt + \cosh kt \quad \dots\dots(12),$$

where

$$\nu = \frac{\mu - \cosh \theta}{\sinh \theta} = \begin{pmatrix} \kappa & -\lambda \\ \lambda & -\kappa \end{pmatrix} \quad \dots\dots(13)$$

and by (10)

$$\nu^2 = 1 \text{ or } \kappa^2 - \lambda^2 = 1 \quad \dots\dots(14).$$

In identifying M with μ^n we are assuming that the surface effects of the sheet may be neglected. Equation (12) shows that

$$\frac{M - \cosh kt}{\sinh kt} = \nu,$$

a matrix independent of t of the form (13). It follows that measurements of the transmission and reflection factors T and R of a sheet of known thickness enable k and ν to be found, and hence the corresponding results for any other thickness can be calculated. The equations to be used are

$$\begin{aligned} k &= \frac{1}{t} \cosh^{-1} \left\{ \frac{1 + T^2 - R^2}{2T} \right\} = \frac{1}{t} \cosh^{-1} \left\{ 1 + \frac{(1-T)^2 - R^2}{2T} \right\}, \\ \frac{\kappa}{1 + \frac{(1-R)^2 - T^2}{2R}} &= \frac{\lambda}{1} = \frac{R}{T \sinh kt}. \end{aligned}$$

If it is desired to avoid measurement of the reflection factor, the simplest procedure is to measure T and T' , the transmission coefficients of sheets of thicknesses t and $2t$. The reflection coefficient R of the former is then given by

$$R^2 = 1 - T^2/T',$$

so that

$$k = \frac{1}{t} \cosh^{-1} \frac{T(1+T')}{2T'}$$

and

$$\frac{\kappa}{1 - T \cosh kt} = \frac{\lambda}{(1 - 2T \cosh kt + T^2)^{\frac{1}{2}}} = \frac{1}{T \sinh kt}.$$

We can now readily express the hiding power h in terms of κ , λ , and kt . We find

$$(P_1 - P_2)h = \{\kappa - \lambda(P_1 + P_2) + \kappa P_1 P_2\} \lambda \cosh 2kt + (1 - P_1 P_2) \lambda \sinh 2kt \\ - \kappa \{\lambda - \kappa(P_1 + P_2) + \lambda P_1 P_2\} \dots\dots(15)$$

or for the absolute hiding power

$$H = \lambda(\kappa - \lambda) \cosh 2kt + \lambda \sinh 2kt + \kappa(\kappa - \lambda) \dots\dots(16).$$

When the value of kt is so large that e^{-2kt} is negligible, we have

$$H = \frac{1}{2} \lambda (1 + \kappa - \lambda) e^{2kt} + \kappa(\kappa - \lambda)$$

and this will frequently be represented with sufficient accuracy by

$$H = \frac{1}{2} \lambda (1 + \kappa - \lambda) e^{2kt} \dots\dots(17).$$

When e^{-kt} is neglected, we obtain $T^{-1} = \frac{1}{2} (1 + \kappa) e^{kt}$, and thus for thick sheets

$$HT^2 = \frac{2\lambda(1 + \kappa - \lambda)}{(1 + \kappa)^2} = \frac{2(1 + \lambda - \kappa)}{1 + \kappa} \dots\dots(18),$$

a constant of the particular material lying between the limits $\frac{1}{2}$ and 0.

We have not yet identified κ and λ with any intrinsic property of the material. Various interpretations may be found, but all are not of equally obvious significance. The one we shall adopt here refers them to the reflecting factor of an infinitely thick sheet of the material. When the thickness is great we have

$$RT^{-1} = \frac{1}{2} \lambda e^{kt}, \text{ or } R = \frac{\lambda}{1 + \kappa} = \left(\frac{\kappa - 1}{\kappa + 1} \right)^{\frac{1}{2}}.$$

Denote this limiting reflection factor by r .

$$\text{Then} \quad \kappa = \frac{1 + r^2}{1 - r^2}, \quad \lambda = \frac{2r}{1 - r^2} \dots\dots(19)$$

so that materials of high reflecting properties (or small absorption) have large values for κ and λ . For a non-absorbing material κ and λ are infinite and equal. As no light is transmitted by an indefinitely thick sheet, the proportion of light absorbed is $1 - r$. We note from (18) that the limit of HT^2 is $2r(1 - r)$. As this is a lower limit, a sheet which does not transmit more than $\frac{2}{3}\sqrt{r(1 - r)}$ of the incident light will be of adequate hiding power. The numerical examples which follow suggest that the factor $\frac{2}{3}$ can often be increased to about $\frac{1}{2}$. The formula suggests that the most transparent sheets admissible are likely to be composed of materials which reflect half the incident light and absorb the rest when the thickness is great.

The fact that the reflection factor of a thick specimen is important suggests that the best practice may be to make all measurements on the reflected light. With non-absorbing specimens the calculations are extremely simple whatever is measured. In other cases the approximation to r is rapid as the thickness increases, so that a satisfactory measurement of this quantity is quite practicable. The other measurement required is the value of R for a thin sheet of known thickness t . A thin film upon a non-reflecting background is suitable for use when the material is paint. The value of k is then to be found from

$$\coth kt = \frac{\lambda}{R} - \kappa = 1 + \frac{2(r/R - 1)}{1 - r^2} \dots\dots(20).$$

This equation will always give k to a suitable degree of accuracy if care is taken to

choose the thin film to give a suitable value to r/R —say 2 or more. As r approaches unity, k becomes large, and a large error in its value is of small significance.

A thin sheet may be only semi-diffusing: to obviate errors due to this cause it would be most satisfactory invariably to measure total reflection factors by illuminating the specimen at an aperture of a photometric sphere. The measurements could well be carried out photo-electrically.

We may note that as tables of hyperbolic functions will be used in calculations according to this theory, it is simplest to use the tables to find κ and λ from r , making use of the equations

$$\kappa = \cosh 2\epsilon, \quad \lambda = \sinh 2\epsilon, \quad r = \tanh \epsilon.$$

Numerical examples

The way in which according to this theory the properties of sheets depend on r and the thickness is best realised from graphs of the quantities we have considered. As examples a non-absorbing, a moderately absorbing, and a rather highly absorbing

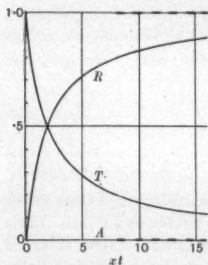


Fig. 1. $r=1$, non-absorbing material

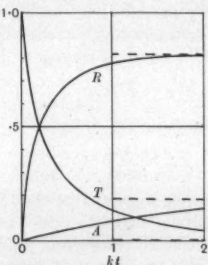


Fig. 2. $r=1/2$, moderately absorbing material

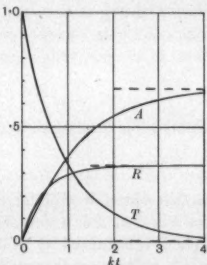
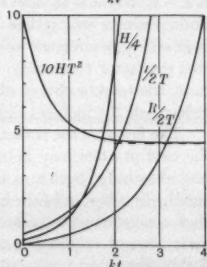
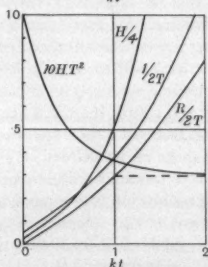
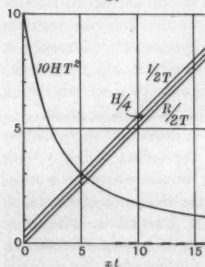


Fig. 3. $r=1/3$, highly absorbing material



In these curves A is the absorption factor, R the reflection factor, T the transmission factor, and H the absolute hiding power. Broken lines denote asymptotes to curves. The A of these figures must not be confused with the A of the text: the latter denotes $1/T$.

medium have been chosen. The values assigned to r are 1, $9/11$, and $1/3$. In addition to the curves themselves their asymptotes have been inserted in the figures. The scales of xt and kt have been chosen in each case to include the features of interest in diagrams of the same size. It will be noticed that, when light is absorbed, R approaches close to its final value r for sheets of the thickness that will be used. In the non-absorbing example the quantities $\frac{1}{2}H$, $1/T$, and R/T are represented by parallel straight lines. In the other examples they are curves which are parallel and disposed similarly to those of non-absorbing materials for small thicknesses, but for greater thicknesses $1/T$ increases much more rapidly than R/T , but more slowly than $\frac{1}{2}H$.

Comparison with experimental results

It is of interest to compare these diagrams with those given by Hanstock. The comparison is of necessity mainly qualitative, but it may be noted that the shapes of the curves for R and T are similar to those obtained experimentally, and the displacements of the curves on the introduction of an absorbing ingredient also correspond. The correspondence is seen in another form in the similarity of the curves for $1/T$. The fact that of the curves we have drawn the hiding power curve lies nearest to $1/T$ is also in accordance with the experimental results. From the nature of the observations involved it is clear that the hiding power curve could not be accurately traced; the sheets with $1/T = 10$ (Hanstock's criterion) are slightly thicker than those required to satisfy the condition $H = 20$, which we assumed as a likely condition. It should be noted that Hanstock's empirical formula for $1/T$ can only give a satisfactory fit to a short length of this curve.

Conclusion

From these experimental results we may reasonably conclude that the theory developed here for dispersive media is in accordance with the experimental facts.

When the contrasts to be obliterated are less extreme than those we have considered the theory may be employed to calculate the thickness of the coat or sheet required.

It has already been pointed out that the theory should be applicable to problems which arise in other industries. Care however must be taken to apply the theory in the right way. For example, in considering the visibility of printing ink through paper, the hiding thickness is not the total thickness of the paper but the thickness through which the ink has not penetrated.

DISCUSSION

Mr R. F. Hanstock: The curves derived theoretically by Mr Smith are similar to those obtained experimentally. While the theoretical consideration is adequate for films having a uniform distribution of diffusing and absorbing particles, I should like to point out that for paint films there is evidence that this condition is not always realised because of the phenomenon technically known as floating of the pigment.

When this takes place, and especially if the paint contains mixed pigments, it is probable that considerable deviation may arise between the theoretical and the experimental curves, particularly between those for the diffuse reflection factor. In practice, therefore, some difficulty may be found in applying the theory to the determination of the hiding powers of paint films.

Mr T. Smith: If it is granted that the theory of simple paints given in the paper is adequate, its extension to mixed paints should not be difficult. Phenomena such as that which Mr Hanstock mentions would be expected, and analysis based on reflection measurements might well prove of decided value in studying them. The analysis already given is applicable to a film composed of alternate strata of different composition (an idealisation of conditions which may be of frequent occurrence), and another case of interest is that in which an excess of one ingredient at one surface changes as we pass through the film to an excess of another at the opposite surface. The correlation of measurements in complex cases with the two basic coefficients for simple films of the separate constituents is not unlikely to lead to conclusions of value.

THE $F/1.8$ QUARTZ MONOCHROMATOR-SPECTROGRAPH

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ABSTRACT. The circumstances in which large monochromators and spectrographs of especially high illuminating power are requisite are discussed and the dependence of illuminating power and instrument efficiency upon the essential optical characteristics of such instruments is investigated. A description is given of a large instrument fulfilling the combined functions of monochromator and spectrograph, the relative aperture of which varies from $F/1.6$ at 0.185 to $F/2.0$ at 1.2μ , and records of performance are described.

Introduction

DEPENDENCE upon wave-length is a characteristic which marks the vast majority of phenomena connected with light in pure optics and in allied subjects. In photochemistry, photoluminescence, photoelectricity, phototherapy, biological research, and similar fields the action of light is found to be inherently a function of its spectral constitution. The fundamental investigation of optical and borderline phenomena consequently always calls for some means of selecting from a heterogeneous radiation a part of that radiation restricted in its spectral constitution to some more or less narrowly limited frequency range. It is possible to accomplish this in appropriate circumstances by comparatively simple means, for example, by means of light filters isolating some more or less broad spectrum band; and if these are employed in conjunction with an appropriate light source a radiation having in favourable circumstances a comparatively high degree of purity may thus be produced.

For general work under ordinary circumstances a slight improvement upon the use of ordinary light filters results from the employment of Lippmann plates as further developed by Ives*, but still the method suffers in any case from its inherent inflexibility, and also from the comparatively low degree of spectral purity which it can afford. In order to increase still further the purity obtained, it is necessary to resort instead to some form of dispersing system with means for selecting and transmitting light from a limited portion of the spectrum, which may then be utilised immediately for the purpose in view or, if demanded by the exigencies of the investigation, may be subjected to a further purification in an analogous manner. This procedure is certainly advantageous in the degree of purity attained, but is invariably and inevitably accompanied by a considerable reduction in illumination. Thus, while the spectroscopic monochromator remains the only satisfactory instrument for this purpose, it will be seen that side by side with its efficiency as a radiation

* H. E. Ives, *Journ. Opt. Soc. Amer.* 1 1917 (49).

isolator and purifier, its illuminating efficiency is scarcely of less importance. This is particularly the case in modern investigations in photochemistry and radiometric research where limited energy of radiation is available and accurate quantitative measurements for different wave-lengths are of considerable importance. For such work a monochromator of the maximum possible illuminating power is essential.

It does not of course necessarily follow that a monochromator should permit of being employed also as a spectrograph; but it is important not to overlook the fact that considerations parallel to those above affecting the illuminating power of monochromators apply likewise to spectrographs. The insistence upon larger light-grasp is evident at the present time also in this field. The spectroscopic investigation of very weak light sources, the study and application of the Raman effect, the spectrographic recording of transient optical phenomena, these and other fields of work call for spectrograph systems of a capacity far exceeding that ordinarily attained. As a further consideration, the inevitable wandering of the image of the spectrum line upon the photographic plate in the course of prolonged exposures, which is a consequence of variation of barometric pressure—if not of temperature—sets a lower limit to the absolute intensity of line recordable with any given instrument, and thus for the recording of faint spectrum lines an instrument of large light-grasp becomes a necessity. The utility of a large-aperture spectrograph or monochromator is accordingly considerably increased whenever the functions of both can be combined in the same instrument, and this has been borne in mind in the design of the present instrument.

The combination so indicated thus resolves itself into a monochromator-spectrograph of high illuminating power, affording as a monochromator appropriate dispersion and purity at the exit slit with accurate mechanical adjustment for variation of the wave-length of the transmitted light; the instrument being as a spectrograph capable also of producing in sharp definition and with high resolving power an extended spectrum image which can be received and photographed upon a photographic plate, the mounting for which may take the position of the exit slit mounting of the monochromator. When employed as a monochromator, such an instrument may be used complete in itself or for the purpose of effecting the final purification in conjunction with other subsidiary means of preliminary purification.

The practical problem thus presented consisted in the production of a monochromator-spectrograph having an illuminating power and linear dispersion the maximum possible compatible with the incidental material limitations, such as the economic necessity of observing some attainable limit in the preliminary requirements—for example, as regards the essential dimensions—in order to maintain the project within practicable bounds. The general theoretical principles conditioning the attainment of a predetermined standard of performance and determining the best form and design of instrument are for the most part to be found in the literature on this subject*, and need not therefore here receive detailed mention. Certain aspects of this question, however, concerning particularly the illuminating power

* Mention may here be made of the following: Pickering, *Phil. Mag.* 36 1868 (41); Rayleigh, *Scientific Papers*, 1 (415 et seq.); Kayser, *Handbuch der Spectroscopie*, Band 1.

and the means of attaining it, have required attention and elaboration in this instance, and these may be considered in the following in some detail.

Illuminating Power and General Design

In the following we shall refer to the components of the spectrograph optical system as (a) the collimating lens, (b) the prism system, and (c) the objective.

It will be seen that somewhat different conditions govern the illuminating power according as the instrument is either (i) used with a medium or *wide* slit, or (ii) applied with a *narrow* slit to sources having *line* spectra requiring definition and resolving power for their investigation. We shall therefore proceed to consider these cases separately. Both cases are treated principally in relation to spectrographs on account of the greater generality there necessary. The former case is that most generally applicable to monochromators.

Case 1. Light emanating from an axial element of the entrance slit aperture of width x and length y falls upon the collimating lens of area A_1 and focal length f_1 at which the entrance pupil of the spectrograph optical system is assumed situated. Suppose the entrance slit to be at medium or wide aperture, and let it be illuminated uniformly by a beam of semi-angular extent sufficiently small to render negligible the second term in the expansion of its cosine, viz. $1 - \frac{1}{2}N_1^{-2}$, where N_1 is the relative aperture of the collimating lens. Consider the performance of the instrument with respect to the axial element $\delta\sigma = \delta x \delta y$ for radiation in the neighbourhood of λ_0 for which wave-length the prism system is set at minimum deviation, this being also the wave-length of the radiation which the objective is adjusted to receive axially. It will be necessary and sufficient to take account of the wave-length range $\lambda_0 \rightarrow \lambda_0 + \delta\lambda$, corresponding to the slit aperture width δx ; in the present circumstances it will be given by

$$\delta\lambda < \frac{\delta x}{f_1} / \left(\frac{dI}{d\lambda} \right)_{\lambda_0} \quad \dots\dots(1),$$

where I is the angle of refraction on emergence of light from the first refracting surface of the prism system, the direction of passage through the prism system being here supposed reversed, while maintaining minimum deviation for λ_0 . The spectrum under investigation may consist of lines or band structure in the neighbourhood of λ_0 , in which case $\delta\lambda$ would be determined by the actual wave-lengths present within the limits implied by (1) treated as an inequality; otherwise, if the spectrum be continuous, $\delta\lambda$ is determined by the relation (1) regarded as an equality.

Subcase (1a). Let the spectrum be continuous and of constant intensity within the wave-length interval $\lambda_0 \rightarrow \lambda_0 + \delta\lambda$. The light flux $\delta\Phi$ passing the slit element in question will then be given by

$$\delta\Phi = E_0 \omega \delta x \delta y \delta\lambda \quad \dots\dots(2a),$$

in which ω denotes the solid angle of the cone of light entering the collimator coaxially and E_0 is a constant characteristic of the specific conditions of illumination.

Subcase (1b). Alternatively, let the spectrum in the wave-length interval

$\lambda_0 \rightarrow \lambda_0 + \delta\lambda$ consist of lines to each of which corresponds a constant value $E_{0\lambda}$, likewise characteristic of the actual conditions of illumination. Then

$$\delta\Phi = \sum_{\lambda_0}^{\lambda_0 + \delta\lambda} E_{0\lambda} \omega \delta x \delta y \quad \dots\dots(2b).$$

In such cases the flux falling upon the entrance pupil of the dispersing system will be respectively, by substitution,

$$\begin{aligned} \delta\Phi &= \frac{A_1 (\delta x)^2 \delta y E_0}{f_1^3 I_{\lambda_0}} & \dots\dots(a) \\ \delta\Phi &= \frac{A_1 \delta x \delta y}{f_1^3} \sum_{\lambda_0}^{\lambda_0 + \delta\lambda} E_{0\lambda} & \dots\dots(b) \end{aligned} \quad \left. \vphantom{\begin{aligned} \delta\Phi &= \frac{A_1 (\delta x)^2 \delta y E_0}{f_1^3 I_{\lambda_0}} \\ \delta\Phi &= \frac{A_1 \delta x \delta y}{f_1^3} \sum_{\lambda_0}^{\lambda_0 + \delta\lambda} E_{0\lambda} \end{aligned}} \right\} (3),$$

where I_{λ_0} is employed in the usual way to denote $(dI/d\lambda)_{\lambda_0}$. The corresponding flux on emergence from the exit pupil of the spectrograph optical system will be $T\delta\Phi$, where T is the transmission factor of the whole system.

The spectrograph is so adjusted that the whole system forms upon the axis of the objective of focal length f_2 an image $\delta x' \delta y'$ of $\delta x \delta y$. This image will be received by the photographic plate in an area $k \delta x' \delta y'$ which is large in comparison with the dimensions of the grain of the photographic plate, k being a factor equal in first approximation to the secant of the inclination of the plate normal to the axis. $\delta x' \delta y'$ is determined by the relation

$$\frac{\delta x \delta y}{f_1^2} = \frac{\delta x' \delta y'}{f_2^2},$$

in which it is assumed that the spectrograph is accurately collimated for wavelength λ_0 , an assumption which is generally approximately fulfilled.

A_1 is closely equal to the area of working aperture, A_2 , of the objective. The surface-density of luminous flux at the image $\delta\sigma'$ of $\delta\sigma$ will consequently be

$$\begin{aligned} s_0 &= \left(\frac{TA_2}{kf_2^2} \right) \frac{E_0 \delta x}{I_{\lambda_0} f_1} & \dots\dots(a) \\ s_0 &= \left(\frac{TA_2}{kf_2^2} \right) \sum_{\lambda_0}^{\lambda_0 + \delta\lambda} \frac{E_{0\lambda}}{\lambda_0} & \dots\dots(b) \end{aligned} \quad \left. \vphantom{\begin{aligned} s_0 &= \left(\frac{TA_2}{kf_2^2} \right) \frac{E_0 \delta x}{I_{\lambda_0} f_1} \\ s_0 &= \left(\frac{TA_2}{kf_2^2} \right) \sum_{\lambda_0}^{\lambda_0 + \delta\lambda} \frac{E_{0\lambda}}{\lambda_0} \end{aligned}} \right\} (4),$$

respectively, to which the illuminating power, speed, and sensitivity of the instrument are directly proportional. From this it may be seen that, considered apart from the nature of the light source, the illuminating power in the case of an extended slit image is dependent upon the quantity $\frac{TA_2}{kf_2^2}$ which will be denoted by s .

Observe that (a) the illuminating power is proportional to the square of $1/N$, i.e. of the reciprocal of the working relative aperture of the objective only, and (b) since the dimensions of the collimator do not enter into (4), except in the dimensionless ratio $\frac{\delta x}{f_1}$, they are not a collateral determining factor.

Case 2. In this case the instrument is assumed to be employed with a fine slit upon line spectra, and it will here be convenient to distinguish between the illuminating power and the ultimate photographic effect, viz. the photometric density actually resulting photographically as the effect of the action of the radiation transmitted by the optical system and concentrated upon the plate. The photographic effect for wave-length λ_0 will in this case be independent of the tilt of the plate, since (a) the Gaussian image $\delta_0 \sigma'$ of the axial slit element $\delta \sigma$, or its projection upon the plate, and (b) the grain of the photographic plate will generally be of the same order of dimensions. The photographic effect of the flux $\delta \Phi$ concentrated upon the image will consequently not be appreciably reduced by the spreading of the image over a somewhat greater area of the photographic plate when the tilt of the plate is moderately increased. The plate will therefore be assumed to be approximately perpendicular to the optical axis of the objective. The remaining factors TA_2/f_s^2 of s will of course continue in this case to affect the illuminating power likewise as in Case 1.

Other effects not above noticed will, however, in this case also play an important part in determining the efficiency of the instrument, viz. (a) the presence of any residual regular or irregular aberration in the optical system; and (b) the redistribution of light within the photographic emulsion; both of these effects leading to lowered sensitivity through the production of a depreciated image owing to the photographic action becoming less localised than in the ideal case.

Ignoring temporarily effect (b), and using unaccented symbols throughout for the image plane, the distinguishing accent being here unnecessary, let it first be assumed that the luminous flux $\delta \Phi$, which in the absence of aberration would be imaged upon $\delta_0 \sigma$, is in fact distributed over the plane $\sigma(x, y)$ in some definite manner so that the flux $d\Phi$ falling upon any given surface element $d\sigma$ in the vicinity of a point (x, y) of the photographic plate is given by

$$d\Phi = f(x, y) \delta \Phi d\sigma \quad \dots\dots(5),$$

subject to the evident condition

$$\delta \Phi = \int_{\sigma} f(x, y) \delta \Phi d\sigma \quad \dots\dots(6),$$

which, as $\delta \Phi$ may be assumed independent of x and y , may be written

$$\int_{\sigma} f(x, y) d\sigma = 1 \quad \dots\dots(7).$$

Equation (5) may be expressed in the form

$$\frac{d\Phi}{d\sigma} = \frac{\delta \Phi}{\delta_0 \sigma} f(x, y) \delta_0 \sigma$$

or, by an obvious change in notation,

$$J = I_0 f(x, y) \delta_0 \sigma \quad \dots\dots(8).$$

Thus, in relation to the point $(x = 0, y = 0)$, the value $f(x, y)_0$ plays the rôle of a proportionality factor controlling the relative intensity at the centre of the image,

and may thus be regarded as representing the relative illuminating efficiency e_1 of the respective optical system. The density D_0 in the centre of the image resulting from this degree of illumination acting for unit time will not, however, be the photographically ideal value assumed determinable on theoretical grounds from the general relation

$$D_0 = \psi(J_0),$$

assuming a definite type of photographic plate and development, and reproduction free from the effects of loss of energy concentration within the emulsion. Owing to the latter effect, which we now take into account, the actual resulting axial density will be D_0' which may be regarded as corresponding to and defining a certain equivalent flux-concentration J_0' , such that

$$D_0' = \psi(J_0'),$$

which, if we let

$$J_0' = e_2 J_0,$$

becomes

$$D_0' = \psi(e_2 J_0) \quad \dots\dots(9).$$

The factor e_2 might be termed the anti-scattering efficiency of the photographic process employed, by analogy with the illuminating efficiency of the optical system as above considered. The possible variation of e_2 with J_0 and with certain other factors of the photographic process is not here excluded, and some variation of this quantity both with intensity and wave-length seems highly probable. It is here unnecessary, however, to pursue the consideration of the subsidiary photographic process in further detail, and for present purposes it will be sufficient to assume for e_2 an average constant value.

As applied to the redistribution of the originally available energy we shall consequently obtain in place of $f(x, y)$ a modified distribution which may be represented by $F(x, y)$ subject to

$$F(x, y)_0 = e_2 f(x, y)_0 \quad \dots\dots(10)$$

for the centre of the image and such that

$$\delta\Phi = \int_{\sigma} F(x, y) \delta\Phi d\sigma \quad \dots\dots(11),$$

that is,

$$\int_{\sigma} F(x, y) d\sigma = 1 \quad \dots\dots(12).$$

If we consider now the effect, upon the developed image, of variation of the size of the Gaussian image of the object to be recorded, it is clear that the dimensions of the total image, assuming a wide slit, may so exceed the limiting dimensions set by the limits of optical and photographic resolution as to render the resulting density at any point not near the margin of the image independent of the imperfections in actual image formation and reproduction, by virtue of the restoration of energy concentration owing to the superimposition, at any point, of contributions

from neighbouring likewise imperfect images. If, however, the extent of the image be comparable with these limiting dimensions the developed density at the nucleus may suffer, with consequent loss of sensitivity to the spectrograph.

In order to gain some approximate quantitative information on this question, we may assume $F(x, y)$ to represent a "normal" symmetrical distribution about the point $(x = 0, y = 0)$, expressible in polar coordinates (r, θ) in the form

$$F(x, y) = H e^{-(ar)^2} \quad \dots\dots(13).$$

Then, by (12), we have

$$H \int_0^{2\pi} \int_0^\infty r e^{-(ar)^2} dr d\theta = 1,$$

whence follows, on effecting the integration, determining H , and substituting in (13),

$$F(x, y) = \frac{a^2}{\pi} e^{-(ar)^2},$$

where $\frac{a^2}{\pi}$, viz. the value $F(x, y)$ assumes for $r = 0$, is thus an efficiency factor for the whole instrument including the photographic plate. This relation will remain constant for Gaussian point images in the neighbourhood of the point $(x = 0, y = 0)$. Thus, considering now the equivalent illumination I at this point resulting from an image of Gaussian radius r_1 , of dimensions small compared with the size of the actual resulting photographic image, corresponding to a value of $(ar_1)^2$ small compared with unity, this will be given by

$$I = \frac{a^2}{\pi} \int_0^{2\pi} \int_0^{r_1} r e^{-(ar)^2} dr d\theta = - \left[e^{-(ar)^2} \right]_0^{r_1},$$

which, by application of a known limit approximation, becomes

$$I \doteq a^2 r_1^2.$$

Thus, besides being proportional to TA_2/f_2^2 , the speed of the spectrograph under such circumstances is also proportional to the efficiency factor $\frac{a^2}{\pi}$ and to the area of the image. Consequently the actual sensitivity of the instrument in the presence of any disturbing phenomena such as those here considered will be less, according as the lines to be photographed are finer and weaker. This incidentally points to the necessity of achieving a standard of optical perfection independent of the seeming performance of the system judged simply by the appearance of the photographic images of the brighter lines, and indicates that one system may be superior to another in speed and sensitivity even when no difference is apparent between them in tests carried out merely in reference to certain more or less prominent spectrum lines.

Considered from the point of view of the general design of the instrument, it follows from the foregoing considerations that the attainment of illuminating power imposes no conditions upon the dimensions of the collimator nor, as far as concerns line spectra, upon the resulting tilt of the focal plane of the spectrograph. The

question as to the actual means of dispersing the light requires in itself, however, some consideration. It is known that, provided the prism system be such as to cause the light to be approximately polarised upon first incidence, the loss of light by reflection does not increase in proportion upon increasing the number of prisms, as each successive prism detracts from the total transmission by a rapidly diminishing proportion. By virtue of this circumstance various writers have concluded that the number of prisms to be employed may be left to be decided entirely by other than optical considerations.

Such reflection loss, however, is in many cases not the only factor to be borne in mind. For, considering the effect upon the linear dispersion at the focal plane of a prismatic system employed with chromatically uncorrected lenses, the increase in dispersion resulting upon increasing the number of prisms from one to two is only about 24 per cent., on account of the incidental reduction in the obliquity of the focal plane. As a result of this the amount of quartz required as raw material in the construction of a two-prism spectrograph system producing the same linear dispersion as a single-prism system with the same illuminating power is actually greater, while of course such further necessary additions as the provision in an adjustable instrument for accurate automatic mechanical adjustment for minimum deviation of the second and subsequent prisms are of important additional economic moment. The joint bearing of material factors such as these, considered in conjunction with the appreciable superiority in the light transmission of the single prism, is in the aggregate distinctly to the advantage of the single-prism system when instruments of large size are under consideration.

Description of the Instrument

A general view of the *F*/1.8 Monochromator-spectrograph is given in Fig. 1, while Fig. 2 shows a view in section. The whole of the mechanical construction is of metal. The framework of the instrument is supported upon a bent girder-form casting of cast iron resting upon three cupped feet (omitted in Fig. 1). The collimator consists of

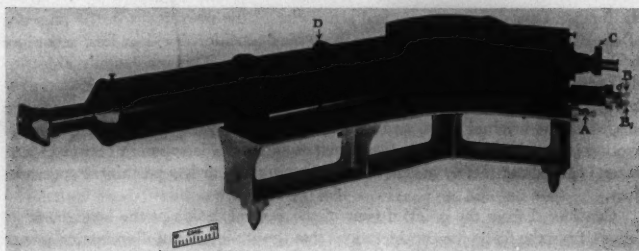


Fig. 1.

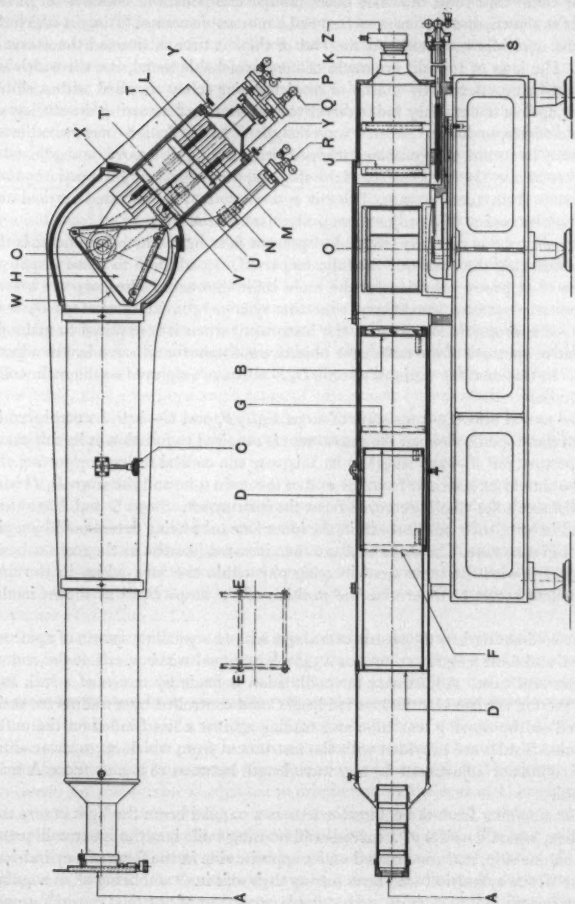


Fig. 2.

a stout compound tube, the main outer tubular component of which is set in two saddles as shown, and carries at its free end a removable conical fitting in which the slit tube is rigidly mounted. At the end of the slit tube is situated the entrance slit *A*. The jaws of this slit are made of non-tarnishable metal, the slit-width adjustment being effected by means of a micrometer screw supplied with a drum, reading against a stationary index direct to 0.01 mm. and actuating the slit jaw on the side corresponding to shorter wave-length. The slit-length may be adjusted accurately by means of a graduated stepped wedge sliding in grooves and adjustable transversely over the front surface of the slit jaws, the wedge being stepped in 5 mm. increments from 10 to 40 mm. The slit as a whole may be adjusted vertical and finally set by means of an adjustment collar fixed to the slit tube.

The slit tube is carefully stopped to prevent false light which in general is the more disturbing the narrower the tube employed. In addition to these stops two systems of stops are provided for the main collimator tube. The purposes served by these stop systems are different: the first system—consisting of stops *D*, *D* in Fig. 2—is appropriate for use with the instrument when it is required to maintain the relative aperture of the collimator objective constant for all wave-length adjustments. In this case the stationary stops *D*, *D* only are employed in the main collimator tube.

The second system—consisting of stops *E*, *E*, *F*, and *G*—is to be employed in place of the stops *D*, *D* when the instrument is required to function at its full available aperture for all wave-lengths. In this case the conical fitting supporting the slit tube should be removed from the end of the main tube and the stops *D*, *D* with their distance tube wholly removed from the instrument. Stops *G* and *F* are then inserted in turn, their position within the inner lens tube being determined by a pin and slot in each case. The stops *E*, *E* are then inserted likewise in the main tube so as to engage with the same fixed locating pin within the tube which in the first arrangement serves to determine the position of the stops *D*, *D* as shown in the figure.

The collimating lens *B* consists of a single lens of crystalline quartz of aperture 11.5 cm. and focal length 111 cm. for λ 5461 Å mounted within a cell at the end of the inner lens tube. Adjustment for collimation is made by means of a rack and pinion motion actuated by the knurled head *C* and controlled by a millimetre scale engraved on the moving lens tube and reading against a fixed index on the outer main tube. Tables are provided with the instrument from which the scale reading for the collimator adjustment for any wave-length between 1850 and 12000 Å may be found.

After emerging from the collimator lens as a parallel beam the light enters the prism box, which consists of a curved-wall housing built into the inner collimator saddle on one side, and constructed on its opposite side in the form of a cylindrical arc as at *X*, the cylindrical wall faces having their axis in *O* and being of an angular extent only sufficient to provide, with suitable excess out of practical considerations, for an efficient light-tight fitting operating over the total range of the prismatic deviation and consequent rotational adjustment of the final imaging system. This

housing is secured by screws to the girder base, and is of course provided with a light-tight cover. The prism is mounted upon a stout hollow metal prism table *W*, its position on this table being defined and permanently maintained by locating guide-pieces which are adjusted and set in position.

The prism *H* itself is of crystalline quartz, has a refracting angle of 60° , is of Cornu form with optically contacted interface, and is of exceptional dimensions, its refracting faces being 17.8 cm. long and 11.4 cm. high. It was corrected by aid of the Hilger prism interferometer, and in the high standard of performance thereby attained represents for its size an optical achievement. It is encased, except over the area of its working aperture, within a metal cover provided with fins and of a form indicated by investigation as being for this type of prism the most suitable for the elimination of serious internal temperature gradients.

The prism table *W* rests upon three feet supported by a lower table which is free to rotate about a central spindle with axis *O*, screwed to the girder base and accurately square with its upper machined surface. It is maintained in position by (a) a pin of circular section, which projects vertically upward from the apex of the lower table and fits a corresponding hole in the bottom of the upper table, and (b) two clamping screws which pass through slots in the upper table and screw into the lower table, thus allowing a small range of adjustment for orientation of the prism. The rigid central spindle passes through a cylindrical fitting within a boss forming an integral part of the lower table and projecting above and below it. The cylindrical interior and exterior surfaces of the upper end of this boss are truly coaxial, the exterior surface thus forming a coaxial central spindle for a fitting ground in a boss cast in the end of the vee-shaped extension to the objective carriage double slide as shown. This slide is located in position by the boss at its end which rests upon the lower rotating table and by its resting upon steel balls as at *S* supported upon the upper machined face of the base and running in vee-grooves formed in a projection on the lower surface of the slide. The slide is thus likewise rotatably mounted with its centre of rotation at *O*.

The rotational adjustments to the position of the slide and prism table about the centre *O*, which are necessary for varying the wave-length of minimum deviation and of the emergent light when the instrument is used as a monochromator, are actuated by the handle *R*, the motion of which is communicated from the screw at *N* to the toothed edge of a quadrant plate rotating about a fixed centre as shown. This rotating quadrant engages with a toothed projection upon the lower surface of the lens slide and thus causes this slide to take up its adjusted position. Simultaneously the prism table is adjusted in orientation by the gear at *U* engaging with the gear at *V*, the former being connected rigidly to the toothed quadrant while the latter is screwed to the under surface of the lower prism table. The gear reductions are so proportioned that the prism table rotates at half the rate of rotation of the lens slide, thus securing the usual conditions of automatic minimum deviation adjustment. The wave-length adjustment is indicated by a scale round the quadrant at *N*, recording whole revolutions of the screw, and by the drum at *R* divided into hundredths of a revolution.

The objective is a two-lens combination, each component being of crystalline quartz. In the case of large aperture systems employed over any appreciable wave-length range it is as important to correct the chromatic variation of spherical aberration as it is to annul the spherical aberration for any given wave-length. For otherwise a lens system which is highly corrected for some particular wave-length band may, on account of the variation in its correction with wave-length, be quite unusable for some adjacent spectral region. It is possible to correct for aberration of this type by appropriate internal design of the optical system and in some optical systems used in spectroscopy the chromatic variation of spherical aberration is actually so corrected. However, the attainment of such correction simultaneously with the fulfilment of other prescribed conditions generally results in increased complexity of the system, a very undesirable feature when it is necessary as in this case to reduce the number of reflecting surfaces and the consequent loss of light to a minimum.

The minimum number of positive lens components for a combination of the required aperture ratio and degree of correction was placed at two. Excluding the theoretical possibility of correction of the chromatic variation of spherical aberration by resorting to extreme forms for the lens components, a combination of this degree of complexity, in which spherical aberration is to be corrected by use of one aspherical surface with negligible coma over the whole field, does not offer the possibility of eliminating the chromatic variation by mere variation of its constructional form. In face of this limitation and in view of the necessity of obtaining the desired correction, the method suggested itself of utilising the separation as a further degree of freedom. Such a degree of freedom is available in any case for the restricted spectrum field of a monochromator. It is likewise applicable to the instrument as a spectrograph on account of this yielding only a limited field at any one exposure.

This method of correction depends inherently upon the assumption, which was investigated and confirmed by rigorous calculation, that it is possible to design a two-lens combination free from spherical aberration and coma as above stated, and so appropriately proportioned as to the distribution of converging powers and air separation, that on passing from 1850 to 12000 Å the spherical aberration introduced by the change in wave-length may be compensated by a mechanically possible variation in the air separation between the components. Provision was accordingly made in the mechanical design of the objective focussing slides for independent motions to the first and second components; and the air separation variation to correct the spherical aberration variation at various wave-lengths was determined and tabulated for the wave-length range 1850 to 12000 Å. Tables were simultaneously prepared for the calculated overall distance from front vertex to principal focus for the same wave-length range in order to provide focussing data determining the absolute positions of the lens carriages for any particular wave-length. The resulting focal length of the objective for 5461 Å is 22.8 cm., the relative aperture varying from $F/1.6$ at 1850 Å to $F/2.0$ at 12000 Å, the spherical aberration being corrected throughout this range by separation adjustment.

Mechanically the appropriate motions were secured by employing two accurately ground dovetail slides parallel to each other, upon each of which a carriage is mounted supporting one of the lens cell-brackets. The bracket supported by one carriage, as at T , overhangs the opposite carriage without making contact with it as shown in Fig. 3, the bracket-cells being thus parallel and coaxial. The position of each carriage along the slide is adjusted by the micrometer screws L and M of 1 mm. pitch, actuated by the adjusting handles P and Q respectively. Each screw is controlled by a Veeder counter recording whole revolutions and by a drum indicating hundredths of a revolution, adjacent to the corresponding adjusting handle. The focussing adjustment of the lens carriages by means of these adjusting drums and counters is made for any desired wave-length by reference to the appropriate

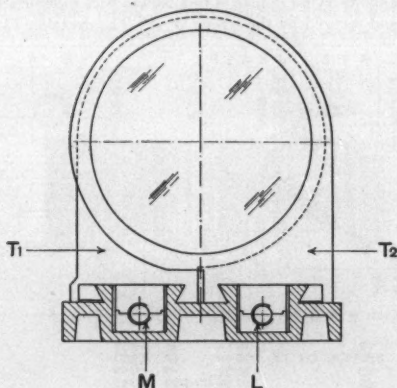


Fig. 3.

tables above referred to. Fig. 3 shows a view of the lens carriages upon their respective slides, as they would be seen from the interior of the camera box supposing the end removed, and illustrates how the lens components are mounted in separate coaxial cells fitted to carriages T_1 and T_2 running upon their slides and each with independent longitudinal screw adjustment by means of the screws L and M .

In the use of the instrument as a monochromator, light of the required wave-length is imaged at the slit Z (Fig. 2) of the thermopile chamber situated at the extreme end of the enclosure housing the objective system above described. This housing is rigidly attached to the lens carriage slides and consequently rotates with these when by rotation of drum and handle R the whole system is reset for some particular wave-length. The inner extremities of the walls of this rotating enclosure or camera box are made in the form of cylindrical arcs about the axis O and concentric with the similarly shaped end of the prism box as above described. A light-

tight fitting between these contiguous surfaces of the prism box and camera box is effected by the use at *X* (Fig. 2) of felt packing. The whole camera box with lens adjustments is thus permitted to take up its position for any required region of the spectrum, and likewise in the use of the instrument as a spectrograph any required spectral region may be brought to the centre of the field by effecting the corresponding adjustments at the drums *P*, *Q*, and *R*.

When it is required to convert the instrument from a monochromator to a spectrograph the screws *Y*, *Y* shown in Figs. 2 and 4 are removed to allow the whole slit end to be withdrawn from the apparatus. In place of this a carrier for a darkslide, constructed to be interchangeable with the removed slit end, is then placed in position and secured likewise by the screws *Y*, *Y* above referred to. The left diagram of Fig. 4 shows a view in plan of this camera back mounted in position upon the instrument and secured by the screws *Y*, *Y*. The darkslide *C* itself fits into a

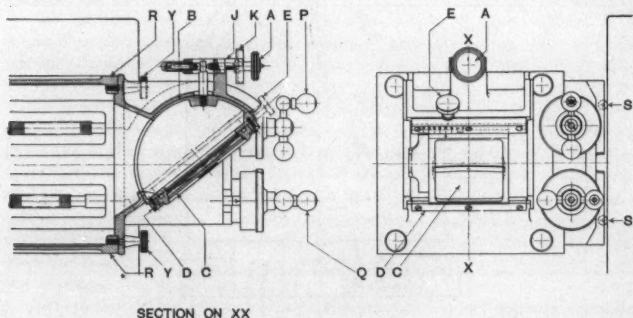


Fig. 4.

vertical slide *D* and, by means of a rack and pinion motion actuated by the adjusting knob *E*, may be adjusted vertically to permit more than one exposure to be given upon one plate. The slide *D* forms part of a hollow hemicylinder which fits as at *B* into an outer hemicylindrical shell having a projection with a square face enabling the whole to be attached to the open end *R* of the instrument. Such a construction enables a rotational adjustment for variation of plate tilt to be given to the darkslide carrier without necessitating the employment of camera bellows. The rotational position of the inner cylindrical member is determined by the position of the adjusting screw *A*, indicated by a drum *K* reading against a scale *J* and divided into hundredths of a revolution. The adjusting screw bears against a pin projecting from the inner member through the outer shell, and is opposed in its motion by a plunger and spring which bears likewise against the opposite surface of this projecting pin. By such means one may set immediately for the correct tilt-adjustment for any desired spectral region.

The position of the darkslide within the stationary slide *D* is indicated by an arrow index reading against a millimetre scale. This is illustrated in the right-hand view of Fig. 4 representing an end elevation of the camera end, projected from the plan shown in the same figure. This elevation being projected from the adjoining plan, the line *XX* is accordingly the trace of a *horizontal* plane. The end of the instrument body is supported upon the two steel balls *S, S* previously referred to, and shown also in this view, resting upon the horizontal upper surface of the girder base of the whole instrument.

Performance of the Instrument

The high illuminating power of this instrument both as monochromator and as spectrograph, and its high standard of definition are evidenced by results of its

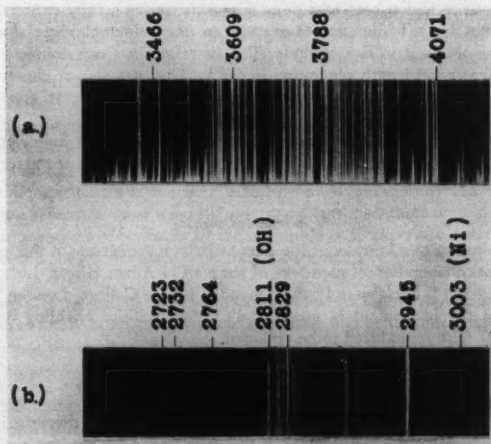


Fig. 5.

performance in Figs. 5a and 5b, these being reproductions, enlarged ten times, of spectrograms which were made using different light sources, as below described. Wellington Antiscreen plates and standard hydroquinone developer to the Wellington formula were employed in both cases, the slit-width being 0.01 mm.

(a) An iron arc running at 2.0 amperes photographed for the region 0.34–0.41 μ . Some little difficulty was at first experienced in this case in obtaining a correctly exposed test-photograph on account of the negatives, obtained in the shortest

possible exposure-time with the available means, resulting considerably over-exposed. Some further control of the photographic exposure as defined by the intensity-time product, It , was obtained by variation also of the factor I by the interposition of an absorbing screen, the transmission factor of which for the spectrum range under consideration was found to be 0.032. The spectrograms obtained in this manner were fully exposed for an exposure-time of 0.01 second, obtained by means of an accurate photographic shutter. The equivalent normal exposure for an unscreened source is thus one three-thousandth of one second. For purposes of comparison it may be mentioned that with the usual laboratory instruments, operating under similar conditions and without the screen, an exposure-time measuring several seconds is necessary for the production of a fully exposed negative.

(b) Secondly, a helium gas discharge lamp taking 8 milliamperes at 200 volts D.C., fitted with a quartz window for work in the ultra-violet spectrum, was employed as a light source of moderately low intensity, special attention being directed to the spectrum region 0.27 to 0.31 μ . With this source a fully exposed negative showing lines due to helium and band structure of considerable detail due to water vapour was obtained in five seconds. The latter shows close doublets of which the minimum wave-length separation is 0.8 Å. This was shown clearly resolved on the plate. The dispersion in this region of the spectrum is approximately 42 Å/mm.

The reproductions unfortunately fail to show some of the finer detail of the original negatives, but nevertheless may perhaps indicate in some degree the high sensitivity and standard of performance of the instrument.

Acknowledgments

It is a pleasure to acknowledge the extent to which the success of this instrument is due to the collaboration of members of the staff of Adam Hilger, Ltd. Grateful acknowledgment is due also to Prof. Allmand of King's College, London, in whose laboratories the instrument is now installed, for facilities enabling one of the photographs here reproduced to be obtained.

DISCUSSION

Mr J. Guild: Messrs Hilger are to be congratulated on their enterprise in designing and producing a new instrument of this type under the present conditions of economic depression. The only proper basis for criticism, either adverse or appreciative, of an instrument developed by a careful designer is actual experience of its use. Without such experience any comments I could offer would be of little value. The evident care with which Mr Perry has investigated the various problems which have arisen in the design seems, however, to warrant confidence that this combined monochromator and spectrograph will behave as satisfactorily as is possible for an instrument designed to fulfil several distinct functions.

Mr T. Smith: In the course of his work on this instrument Mr Perry has attacked a new optical problem. Variable separation between component lenses has

been adopted for various purposes in other instruments, but as far as I know this is the first occasion on which it has been used to correct spherical aberration at different wave-lengths. It would be of interest if the author could state the magnitude of the outstanding errors on the axis in terms of wave-lengths for the useful range of the instrument, also the outstanding coma in wave-length units over this range, the angular field over which the image is good, and the nature of the aberration which is first effective in causing deterioration of the definition. It is particularly pleasing to see that theoretical designs employing non-spherical surfaces are being evolved at Messrs Adam Hilger's, where the interferometers of such great value in the practical attainment of such forms were developed and first used. Mr Perry is much to be congratulated on the successful outcome of this very interesting investigation.

Mr Perry: I thank Mr Guild and Mr Smith for their remarks. For a parallel beam of wave-length 0.546μ incident axially upon the objective, the spherical aberration is zero for all zones, while for other wave-lengths, marginal spherical correction longitudinally at the focus is accompanied by a zonal residual. This residual higher spherical aberration, expressed as a relative retardation referred to the paraxial beam on emergence from the objective, for the margin of the aperture, i.e. semi-diameter $H = 57.5 \text{ mm.}$, is -0.051 and $+0.533 \mu$ for wave-lengths 1.16 and 0.214μ respectively, positive retardation being true retardation and signifying what is commonly known as spherical *over*-correction. Expressed in wave-lengths of the corresponding radiation, these retardations are -0.044 and 2.49λ respectively, and are stationary values at the semi-diameter above mentioned. The departures from a mean reference wave would of course be less than these amounts. Effects upon these quantities of terms in H^3 are negligible at this aperture, and the relative retardation at a chosen reference wave may consequently be determined from these data.

The coma for an originally incident beam of wave-length 1.16μ at an obliquity of 1° to the axis may be represented as a retardation $\eta_{1.16}$ in a manner analogously to the above, $\eta_{1.16}$ being given by

$$\eta_{1.16} = -0.0088356 \sin^3 \theta + 0.010932 \sin^4 \theta - 0.045628 \sin^5 \theta - 0.18477 \sin^6 \theta,$$

where θ is the semi-angular aperture subtended from the paraxial focus, the aberration being an inward coma. For $\sin \theta = \pm 0.24$, viz. the margin of the aperture, $\eta_{1.16} = -0.00221$ and $+0.00405 \mu$. For wave-length 0.185μ , the farthest possible limit of transmission of the instrument, the objective is affected by outward coma which relatively to the wave-length is substantially greater than these amounts, viz. $\eta_{0.185} = 0.00451$ and -0.00351μ . However, the optical material would not allow of transmission over the major portion of the prism aperture at this wave-length.

Photographically the total field of the objective is limited to 2° by its curvature of field, which is the more important because of the somewhat high aperture ratio. The simplicity of the objective construction should of course be borne in mind in this connexion. This extent of field is sufficient for the purposes for which the instrument is designed.

A PHOTOMETRIC EYEPiece FOR VISUAL QUANTITATIVE SPECTRUM ANALYSIS

By F. TWYMAN, F.INST.P., F.R.S.

(Messrs Adam Hilger, Ltd.)

MS. received, 8th March, 1932. Read and discussed, 14th April, 1932.

ABSTRACT. A new type of eyepiece is described whereby a spectrum is split into halves, a selected line of the one half being brought vertically above a selected line of the other. The intensity of the brighter line is then reduced by means of a neutral tint wedge until equality of the two intensities is obtained. The eyepiece is used in an analysis of some cadmium-lead alloys, an accuracy of some 11 per cent. of the cadmium content being obtained. Of the error 7 per cent. is due to irregularities of the wedge used.

It has been established that, given a means of photometric comparison of the intensity of spectrum lines, and a set of samples which contain various known percentages of a metallic constituent, but in other respects identical or only differing slightly, it is possible in the case of metals, alloys, powders, and liquids to estimate the metallic impurity spectrographically with an error of the order of 10 per cent. of the metallic constituent sought*. It has recently been shown that it is not always necessary to have a number of specimens of known content, but with alloys and solutions it often suffices to have two such specimens (Twyman and Hitchen, *loc. cit.*).

There are a number of means of photometric comparison to choose from, and those inclined to consider the possible utility to them of spectroscopic analysis may be rendered undecided by this very great choice. It is thus with some reluctance that I bring forward still a further instrumental means of comparing the intensities of spectrum lines. It is distinct from most previous ones, however, in being appropriate for visual spectroscopy.

Quantitative spectrum analysis involves two main problems:

- (1) How to produce from a substance a spectrum which will truly represent it.
- (2) How to compare the intensities of selected lines in such a spectrum.

The intensity of a spectrum line, whether as seen direct by the eye or as represented in a photograph, is determined by such a variety of circumstances—method of excitation, dimensions of the spectroscope, nature of the element, and the wave-length of the lines—that it is impossible to predict *a priori* what intensity

* F. Twyman and A. A. Fitch, "The quantitative analysis of steels by spectrum analysis," *Journ. Iron and Steel Inst.* 129 1930 (289); F. Twyman and C. S. Hitchen, "Estimation of metals in solution by means of their spark spectra," *Proc. Roy. Soc. A*, 133 1931 (72); D. M. Smith, "The spectrographic assay of some alloys of lead," *Journ. Inst. Metals*, 46 1931 (114).

of lines will correspond with a given percentage of a metal in a substance. The spectroscopist is therefore on safe ground only if he maintains his technique constant throughout any series of observations in which he desires to correlate the intensities of spectrum lines with the percentages present of the corresponding metals.

To this must be added that common sense and experience alike suggest that to secure that the comparisons of intensity are made under like conditions, they should be made between a line of the element sought and a line of the dominant element in one and the same spectrum. This method is now generally adopted by spectroscopists in quantitative analysis, being known as "Gerlach's Internal

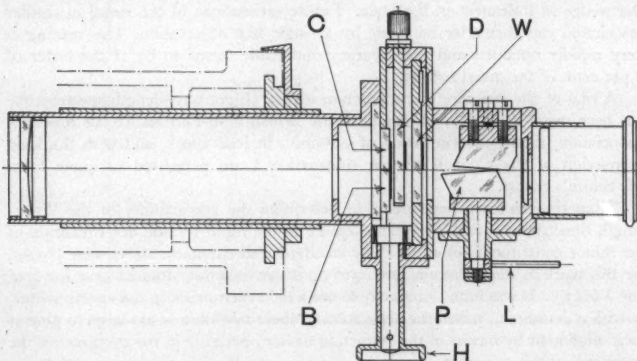


Fig. 1.

Standard" method*. The eyepiece† now to be described makes use of this principle in a very simple way for visual spectrum analysis of alloys‡.

This eyepiece (see Fig. 1) may be fitted into any visual spectrometer which is of sufficient dispersion for the adequate separation of the lines. By rotation of the prism *P* by means of the lever *L*, a selected line of the metal sought is brought into juxtaposition with a neighbouring line of the dominant metal. These two lines should, of course, be as near together in the spectrum as is possible, provided they are both of an appropriate character. A neutral tint wedge *W*, extending over one-half of the field and movable by rack and pinion by turning the head *H*, is so adjusted that the intensity of the stronger line is reduced to equal that of the weaker

* For a full discussion of the method, see W. Gerlach and E. Schweitzer, "Chemical analysis by the emission spectrum."

† British Patent 334,618, June, 1929, and corresponding foreign applications.

‡ It is probably also applicable to powders and liquids.

one and the density of the wedge necessary to produce this matching is indicated on a scale. D is an auxiliary density of value 1.0 which may be brought into operation behind the neutral tint wedge W to increase the density range when required. C is a compensating wedge, giving even density across the upper half of the field in which the neutral tint wedge W is situated, and B is a balancing density placed in the lower half of the field to provide that both are of equal intensity when the neutral tint wedge W is in its "zero" position. The effective range of density* of W is from 0 to 2.0.

The matching process being carried out for specimens of known percentages, and the scale readings recorded, a graph is drawn connecting the percentage or, preferably, the logarithm of the percentage with the reading of the position of the wedge as indicated on the scale. Future estimations of the metal in similar association can thereafter be made by a single like observation. The reading is very rapidly obtained and the accuracy obtainable seems to be of the order of 5 per cent. of the metal sought.

A trial of the eyepiece in conjunction with a Hilger wave-length spectroscopy has been made by Mr D. M. Smith of the British Non-Ferrous Metals Research Association, in the determination of cadmium in lead alloys, and with the kind permission of the Council of that Association I am permitted to quote from Mr Smith's report.

"Experiments have been made to determine the applicability of the Wave-length Spectrometer and Twyman eyepiece to the rapid routine determination of the minor constituents of alloys. The standard lead-cadmium alloys were chosen for this work as the cadmium line $\lambda 5085.9$ is conveniently situated near the lead line $\lambda 5005.5$. It was found necessary to use a light melt prism in the spectrometer, as with a dense melt prism the separation of these two lines is too large to permit their alignment by means of the refracting device operating in the eyepiece on the lower half of the spectrum.

"Owing to the proximity of the air lines $\lambda 5005.1$ and $\lambda 5001.3$ (due to nitrogen), self-inductance was inserted in the secondary circuit to suppress their appearance. It was found more satisfactory to work without a condensing lens and to use a fairly wide slit. (It is easier to compare the relative intensities of bands than of sharp lines.) For the production of the spark, the standard outfit was used, having 0.005 mF capacity and 0.25 millihenry self-induction. The spark (1/8th of an inch in length) was placed at a distance of 10 cm. from the spectrometer slit.

"When the 'note' of the spark changed, or when there were any irregularities in the running of the discharge, the relative intensities of the lines showed variations. Hence the measurements were only made when the discharge was running normally.

"For the purposes of preparing a graph, the readings of the wedge were taken as positive when the upper line (viewed through the neutral wedge) was the cadmium line and the negative when the lead line was uppermost. This was

* Density = $\log_{10} (I_0/I)$, where I_0 is the intensity of the incident and I that of the transmitted radiation.

necessary as equality of intensity between these two lines occurred at about 0.3 per cent. Cd."

The results obtained are shown in the accompanying table, and in Fig. 2 is shown a plot of the scale reading against the logarithm of the percentage content. Before discussing these results, however, it is necessary to indicate the relationship existing between the density D of the wedge (as defined earlier) and the scale reading S of the eyepiece.

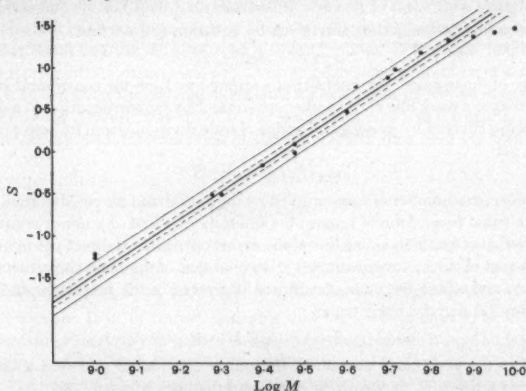


Fig. 2. Variation of S with $\log M$.

Alloy	% Cd	$\log \% \text{ Cd}$	Scale readings	Mean S
LC 10	1.0	10.000	1.5, 1.5, 1.6, 1.5	1.5
LC 9A	0.9	9.954	1.4, 1.5, 1.5, 1.4	1.45
LC 8	0.8	9.903	1.4, 1.3, 1.4, 1.4	1.4
LC 7A	0.7	9.845	1.4, 1.3, 1.4, 1.3	1.35
LC 6	0.6	9.778	1.2, 1.3, 1.1, 1.2	1.2
JTW	0.52	9.716	1.1, 1.0, 0.9, 0.9	1.0
LC 5A	0.5	9.699	0.9, 0.9, 0.9, 1.0	0.9
JTV	0.42	9.623	0.7, 0.9, 0.8, 0.8	0.8
LC 4	0.4	9.602	0.5, 0.6, 0.5, 0.4	0.5
JTU	0.3	9.477	0.0, 0.0, 0.1, 0.0	0.0
LC 3A	0.3	9.477	0.1, 0.2, 0.1, 0.0	0.1
JRS	0.25	9.398	-0.2, 0.2, 0.1, 0.1	-0.15
LC 2	0.2	9.301	-0.5, 0.4, 0.6, 0.5	-0.5
JTT	0.19	9.279	-0.4, 0.5, 0.4, 0.6	-0.5
LCIA	0.1	9.000	-1.2, 1.2, 1.25, 1.15	-1.2
JTS	0.1	9.000	-1.2, 1.3, 1.3, 1.2	-1.25

Subsequent to the investigation made by Smith the wedge was calibrated for wave-length 4950 Å by means of a Hilger-Nutting polarisation photometer. It was

found that the density D was directly proportional to the scale reading S , the relation subsisting between the two quantities being

$$D = 0.37 S \quad \text{.....(1).}$$

Over the region employed by Smith in no case did the density, as estimated from this formula, exhibit a discrepancy of more than 0.04.

The density D has already been defined as

$$D = \log_{10} (I_0/I) \quad \text{.....(2);}$$

hence, substituting for D the value given by equation (1), we find

$$0.37 S = \log_{10} (I_0/I) \quad \text{.....(3).}$$

In the use of the eyepiece the intensity of a strong line I_L of one constituent is made equal to that of a weak line I_C of another constituent by the introduction of a density D , this being effected by moving the wedge S scale divisions from its zero position. We thus get

$$\log_{10} (I_L/I_C) = 0.37 S \quad \text{.....(4).}$$

Furthermore, in a number of cases studied by the author and his collaborators, it has been found that (over definite ranges) the intensity of a line of a minor constituent relative to that of a neighbouring line of the main constituent is directly proportional to the content of the minor constituent relative to that of the main constituent. For such cases, and where the main constituent is present in the preponderant degree in question, (4) may be rewritten as

$$S = 2.7 \log_{10} M - B \quad \text{.....(5),}$$

where M is the percentage content of the minor constituent and B is a constant depending on the alloy under investigation and the lines selected.

Returning to Fig. 2, it will be seen that over the range of 0.15 to 0.7 per cent. Cd the relation between S and D is as given by equation (5). The deviations from linearity observed at the extremes of the range are very similar to those that we have observed in a number of other cases.

Smith notes that: "It should be possible, with practice, to obtain a mean of 4 or 5 readings correct to half a division on the scale (i.e. to 0.05)." Were this the only error involved, then its final effect (considering only the straight portion of the graph) would be to produce an uncertainty of 4 per cent. in M . On a logarithmic plotting such as we have employed this may be shown by the two broken lines in Fig. 2, which are placed at such a distance from the graph that any point lying within the area marked by the broken lines is within 4 per cent. of its correct value. It is at once obvious that the errors of the measurements are actually greater than this. The measurement of the density, as we have stated above, showed that it may be in error by 0.04. This, as will be seen from the equations, is equivalent to an error of 0.10 in S . The total error possible is thus equivalent to 0.15 in the scale reading. In terms of M the error will then be 11 per cent. The *thin* full lines shown in the graph lie at this distance from the main line, and it will be seen that, with the exception of the points at the ends, the maximum error is of the order we have indicated.

In conclusion then, one may state that given a perfect wedge it is possible to match lines with such an exactness that an accuracy of 4 per cent. in the percentage content is obtainable, while in the instrument described imperfections in the wedge (combined with the error in matching the intensities) are such as to yield a final accuracy of 11 per cent.

Note added in proof. The wedge in the instrument used by Smith was a circular one and it is known that such wedges are more difficult to make than straight ones. An examination has since been made of a number of straight wedges and it has been found that, assuming the density to be a linear function of the scale reading, the maximum error in the density is 0.015. This is equivalent to an error of 0.04 in S , whence it follows that the maximum amount the scale reading can be in error (due to the combined effect of inaccuracy in the wedge and errors in the setting) will be 0.09. The accuracy of determination of the content will then be 8 per cent.

DISCUSSION

Prof. A. O. Rankine: What is the maximum wave-length interval between the spectral lines whose intensities are compared? Is it so large that there is a noticeable colour difference, and, if so, does this not make the matching difficult?

Mr Twyman: It is, of course, desirable that the spectral lines to be compared should not differ markedly in colour, and as a very rough guide it may be taken that lines within 100 Å of each other can be dealt with.

The eyepiece, up to the present, has chiefly been used for examining alloy steels in which there is an ample number of iron lines to permit of this condition being fulfilled. It will be noticed, however, that in the paper by D. M. Smith, to which reference is made, lines were compared which show a considerable colour difference (Cd 5085.9 Å and Pb 5005.5 Å) and where the spectrum of the main substance consists of few lines this is often unavoidable.

Those accustomed to the photometry of different coloured radiations acquire a confidence in reproducing matching, even when the photometric fields to be matched differ greatly in colour and it is hoped, therefore, that in the use of the eyepiece it will be found that consistent results are obtained, even where the colour of the spectral lines differs considerably.

The adjustment actually provided in the eyepiece is sufficient to bring together differently coloured parts of the spectrum, except in the violet region below 4500 Å, where the dispersion of the spectroscope is sufficiently great to obviate the colour change.

OBITUARY NOTICE*

EMERITUS PROFESSOR ARCHIBALD BARR, D.Sc., LL.D., F.R.S.

By the death of Prof. Archibald Barr on 5th August, 1931, the Optical Society lost one of its most distinguished Fellows and Past Presidents.

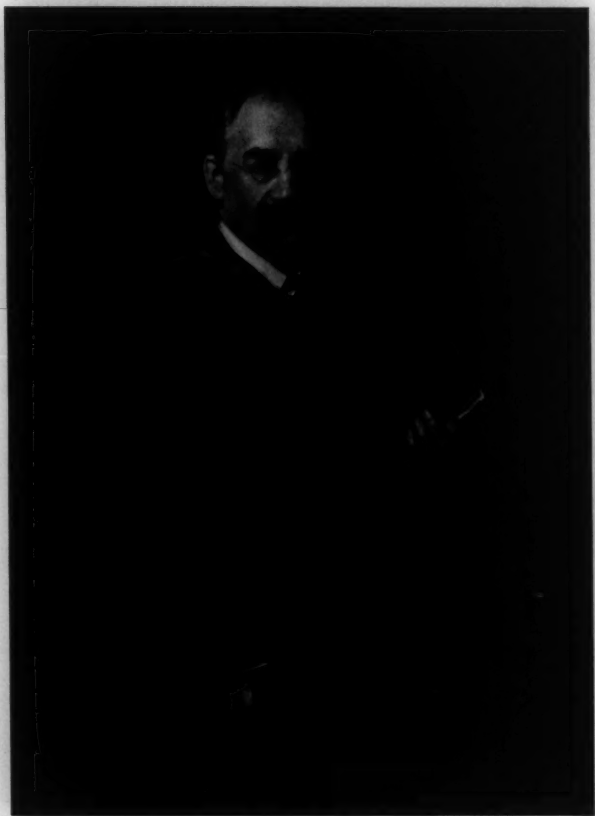
Prof. Barr was born in Paisley in 1855, and received his early education in the Grammar School of that town. He afterwards studied at Glasgow University, where he graduated in engineering science at the age of 21. His good work at the University resulted in his appointment as assistant to Prof. James Thomson. In 1884 he was appointed Professor of Mechanical Engineering in the University of Leeds (then known as the Yorkshire College), and in 1889 he succeeded Prof. James Thomson as Regius Professor of Engineering and Mechanics at his old University. He held this chair until 1913 when he resigned.

The two outstanding results of his sojourn at Leeds were the organisation of the Engineering Laboratory there which rapidly became one of the most important in the country, and his friendship with Dr Stroud, Professor of Physics in the same College. This friendship led eventually to the foundation of the great firm which bears their names.

During his tenure of the chair at Glasgow, Prof. Barr did not spare himself in his endeavours to create an Engineering Department in keeping with the importance of Glasgow and the Clyde, then, as now, the greatest engineering centre in Scotland. The James Watt Engineering Laboratories can truly be said to owe their existence to Prof. Barr's insight and industry. During the 25 years he occupied this chair the number of engineering students increased sixfold, and the prestige of his department grew rapidly.

In 1888, Prof. Barr and his friend Prof. Stroud saw an advertisement by the War Office asking for designs for a rangefinder which could be carried by a man in the field, and had to fulfil several stringent conditions. They became interested in this intriguing problem, and in a few weeks had produced the essential design of the Barr and Stroud rangefinder. From that time they may be said never to have ceased to develop and perfect these instruments. Many parts of the first instruments were constructed by Dr Barr himself in his private workshop in Glasgow, and the instruments were tested and adjusted on the roof of his house. In 1895 an old stable was converted into the first factory of Messrs Barr and Stroud, and, as the demand for their rangefinders grew, newer and larger premises were obtained, until in 1904 the first portion of the great factory at Anniesland was built. This factory was extended very rapidly during the war when some 2000 workmen were employed. At the present time it is the only factory in the country where all stages in the manu-

* Summary of an appreciation delivered by Mr W. B. Coutts at the meeting held on 15th October, 1931.



Portrait by G. Fiddes Watt, R.A.

Photo by Annan & Sons, Glasgow

EMERITUS PROFESSOR ARCHIBALD BARR, D.Sc., LL.D., F.R.S.

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facture of large optical instruments are carried out from the raw materials to the finished article. Even the optical glass is made in the factory.

Prof. Barr was a most able organiser as well as an inventor and engineer, and he took the keenest interest in the development of the business from its inception until his death. He did not confine his attention to the technical side, but initiated schemes for the well-being of his employees during their hours of work and recreation, and schemes for encouraging output.

In addition to the rangefinders by which he is best known, Prof. Barr developed many other military and naval instruments, including fire control systems, bomb dropping sights for aircraft, submarine periscopes, etc. His post-war work included the development of the Optophone and apparatus for the production of maps from aerial photographs.

The University of Glasgow conferred the degree of Doctor of Laws upon him in 1914, in which year he was elected a Fellow of the Optical Society. The Society conferred its highest honour upon him by electing him President in 1923, and in the same year he was elected a Fellow of the Royal Society. During his tenure of office as President, business activities prevented him from attending as many meetings as he would have liked, but, apart from this, he allowed nothing to stand in the way of his service to the Society, ever holding himself ready to give advice or make decisions, and keeping in close touch with the Society's activities. Members who were present at the February meeting in 1925 will remember the inspiring Presidential Address* he delivered to the Society.

Prof. Barr will be remembered as a great engineer and teacher by the world at large, but by those who knew him he will be remembered above all as a great gentleman. His gifts of rapidly reducing problems to their essential principles, and of giving logical and lucid explanations of complicated processes and mechanisms, will long be remembered by his students and those who consulted him on technical matters. He possessed a fine sense of humour, and a ready sympathy with others. His courtesy, friendliness, and fairness endeared him to all.

* *Trans. Opt. Soc.* 26 1924-25 (121).

REVIEWS

A Treatise on Photo-Elasticity. By Prof. E. G. COKER and Prof. L. N. G. FILON.
Pp. xviii + 720; 332 Figs. and 14 Coloured Plates. (Cambridge: University Press, 1931.) Price 50s. net.

The stresses to which steel, timber, building materials, glass and many other substances are subjected when incorporated in structures, machines, and instruments, and the strains they undergo, are obviously matters of first-rate importance to the engineer, the designer, and the scientific worker; indirectly, through their influence on the safety as well as the cost of great public and private undertakings, they are not without importance, even if of no intellectual interest, to all of us. The direct determination of stresses in actual engineering structures is in general a matter of the greatest difficulty; frequently it is impossible. Nevertheless their measurement, indirectly if not directly, is essential for the proper advancement of the great branches of applied science. One of the most interesting ways in which it has been sought to gain this knowledge depends upon the double refraction produced in homogeneous transparent materials such as glass and celluloid when they are strained. As is well known the authors of this book have for a long period been engaged in researches on this subject which, largely as a result of their investigations, have led to conclusions of the greatest interest to the engineer. The ever growing importance of more precise knowledge, and the expanding field to which this method of enquiry has proved applicable, have led to the appearance of this authoritative treatment of the foundations on which this branch of science has been built and of examples illustrating alike its power and its limitations. It will not be disputed that the writers of this book are almost uniquely qualified for the task they have now completed.

The book will inevitably prove of much interest to large numbers of physicists, but it has been written more particularly for engineers. To follow the investigations included here, some will merely require to study the parts dealing more particularly with the optical side of the subject; most will need in addition to extend their acquaintance with elastic theory. The authors have wisely made references to other books unnecessary by including the whole of the fundamental theory in the opening chapters. The first of these deals with the refraction of light through crystalline media, the second with the elementary theory of elasticity, and in the third, which is entitled the theory of artificial double refraction, the essential connections between stresses and the optical effects observable in polarised light are developed. Each of these chapters runs to about a hundred pages; none is easy reading, but the treatment is full and no reader capable of following fairly elementary mathematical work with any facility should experience any insuperable difficulty. The details of the mathematical work are arranged to be most readily followed by such readers—we imagine that a more concise treatment, involving the use of standard methods of solving the equations, would usually be preferred by mathematical students. The later chapters deal with numerous applications of the theory in conjunction with measurements made on models. Chapter IV, the longest in the book, is limited to two-dimensional problems involving straight and circular boundaries. Some of the results obtained are of course applicable without gross errors to three-dimensional bodies. The scope of this chapter will be appreciated by mentioning a few of the subjects discussed—cylinders, rings, compressed-gas containers, cutting tools, milling tools, elliptical links, rollers, and several others. The next chapter deals with beams, a feature of particular interest being the discussion of notched beams under flexure. Multiply-connected plates are considered in Chapter VI, entitled

holes and cracks; the variations in stress due to a load applied at any point of an elliptical hole is one of the problems considered. The authors have many interesting observations to make about existing standard tests in Chapter VII, on the testing of materials. The last chapter, entitled structures and machines, is at least as interesting as any other in the book. The problems discussed include many of outstanding importance to the builder, the architect, the civil engineer, the mechanical engineer, the naval constructor, and others. Toothing gearing, nuts and bolts, fused joints, the comparative merits of different methods of securing tyres on locomotive wheels are among the examples taken by the authors. Throughout the book the text is supplemented by a very large number of carefully drawn diagrams. There are also several photographs illustrating the variations in brightness over the field observed in experimental work, and 16 beautifully reproduced figures in colour showing how the stress distribution in typical problems first manifests itself in the laboratory. The volume includes a bibliography of about 14 pages, and name and subject indexes. It goes without saying that the Cambridge University Press has exercised its usual care in the printing and accurate rendering of the text. The book should prove of great value to many groups of workers, and the price, for a book of this character and size, is not unreasonable.

T. S.

Lezioni di Ottica Geometrica. Per GINO GIOTTI. Pp. 470; 246 Figs. and 4 Plates. (Bologna: Nicola Zanichelli, 1931.) Price Lire 70. (Publication No. 3 of The National Institute of Optics, Florence.)

There appears to be a decided preference among teachers in this country to adopt a purely geometrical treatment of the theory of optical instruments. The course described in this book is an excellent example of a detailed exposition on these lines, and is as well worthy of study by those who prefer to follow another route as by those who take the author's path. The book is particularly rich in diagrams, many of which exhibit in an original and forcible way relations usually presented in a different manner. In general the treatment is fuller than is to be found in comparable English books. On the other hand aspects of lens theory which would usually be included in more condensed treatises are here lacking, the reason being that this volume is only one of a series designed, in combination, to cover ground more usually traversed, in much less detail, in a single volume. The same circumstance may explain the introduction of a fresh concept of which no subsequent use is made in this book, e.g. a quantity expressing the shape of a lens.

The principal optical instruments and their properties are fully explained on the principle of the rectilinear propagation of light, but, in accordance with the plan of this series, problems of design, and instruction in calculations of the type involved in their design, are omitted. These are to be treated in another volume in course of preparation. These omissions, together with the absence of numerical examples, tend to give the book a distinctly academic tone, and this effect is rather accentuated by the almost complete absence of any references to modern developments. To take an example, Gullstrand is quoted when the structure of the human eye is being described, but his projective theory of imagery, which we might have expected, particularly in the hands of an Italian writer, to be developed in an attractive geometrical form, is not mentioned. The great merit of the book lies in the excellent and thorough way in which the geometrical treatment of conventional elementary geometrical optics is presented.

The author is conscious of the varying names associated in different countries with leading theorems, and in some cases he provides a key to the more frequent usage. English readers will be surprised at the absence of Snell's name. More space in a volume of this size might with advantage have been given to the history of the subject. Some acknow-

ledgments also seem to be called for, e.g. to Messrs T. Cooke and Sons in respect of Plate IV, which is a reproduction of several of the diffraction patterns contained in the frontispiece to their book "On the Adjustment and Testing of Telescope Objectives."

The book is provided with an index, and is stitched in a paper cover.

T. S.

Prism Size and Orientation in Minimum Deviation Refractometry. Bureau of Standards Research Paper No. 262. By L. W. TILTON. Pp. 18; 6 Figs. (Washington: Superintendent of Documents, United States Department of Commerce, 1931.) Price 10 cents.

In the measurement of refractive index by observations upon prisms at minimum deviation, the accuracy attained in any given instance is generally known to be dependent upon various factors, some of which, such as the essential dimensions of the means employed, are largely within the control of the observer, while others, of which the degree of homogeneity of the material is one of the most important, remain for the most part beyond control. The modern advances in the technique of heat-treatment of glasses enabling a much higher standard of homogeneity to be achieved than formerly have, in the opinion of the author, removed these limitations, and thus render appropriate an enquiry into the effect of the inherent limitations of goniometry upon the ultimate accuracy of determination of refractive index. Such an investigation is prompted by a knowledge of the considerable degree of consistency which is found in goniometry when comparatively small prisms are used.

Commencing with a consideration of the limit of precision, m , of angular measurement by telescope and its relation to the limit of angular resolution, r , the relative magnitude of these constants represented by the factor $K = m/r$ is discussed with reference to experimental data obtained from observations made with various shapes of apertures. Considerable variation in K is found; the values do not however in any case exceed 0.1. The effect of variation in wave-length or of actual aberration is not investigated although various conflicting references are made to the aberrational correction of the objectives employed. The lack of precise information on this point detracts considerably from the utility of the data recorded in this connexion. The author adopts $\frac{1}{25}$ as a suitable typical value for K .

In the three sections that follow, the author considers for a stated accuracy of $\pm 1 \cdot 10^{-6}$ in refractive index the interdependence of the various variables entering into the whole determination, including the number of observations and its effect upon the resulting precision, and expressions are given relating the defining quantities for this assumed accuracy.

The precision in prism orientation necessary to ensure such accuracy is determined, and by the application of elementary error theory to determine the effect of the residual inaccuracy inevitable in the observed deviation the author is able to tabulate a correction to be applied to the refractive index dependent upon the probable error in prism orientation, and arising through the asymmetrical character of the errors in deviation.

The deductions are supported where necessary by reports upon practical tests in which the type of distribution of errors in orientation is investigated and the magnitudes of the residual errors, possibly typical, but applying specifically to the author's particular case, are determined.

The prism size is considered lastly in relation to its effect upon the accuracy of the observed deviation, the conclusion being reached that, in relation to this particular source of error, the length of face of a 60° prism need not exceed 2 cm. in order that the error in refractive index shall be less than $\pm 1 \cdot 10^{-6}$.

It should be emphasised in this connexion that the present paper constitutes an analysis of some errors of minimum deviation refractometry, certain of the important

contributing factors being thus considered singly. The effect of synthesis is not here attempted, and it is important to bear in mind that the supplementary conditions of practical observation must necessitate the dimensional values here found being regarded merely as irreducible minima which in some circumstances will probably require to be considerably exceeded. The paper raises important and interesting questions and, although its phraseology is not very clear in places, the information given is susceptible of generalisation, so that workers in this field should be aware of this publication.

J. W. P.

Quartz Resonators and Oscillators. By P. VIGOUREUX, M.Sc. Pp. 217; 125 Figs. (London: H.M. Stationery Office, 1931.) Price 7s. 6d. net.

This book is well worth the very modest sum of 7s. 6d. asked for the information contained in its 217 pages. Indeed, the bibliography alone, containing references to no less than 186 papers, should be worth this to the hard-pressed worker who is delving into this subject for the first time.

The book is divided into five chapters—General Physical Properties of Quartz; Theory and Application of the Static Piezo-Electric Effect; The Piezo-Electric Resonator; The Quartz Oscillator; and The Structure of Quartz.

It is very fitting that the first and last chapters of a book with this title should be devoted to the properties and structure of quartz as, for practical purposes, this is all-important, and those who have given practical results hitherto have generally omitted to mention any of the tests applied to the quartz to see if its structure was abnormal. The theory of the modes of vibration of plates of various shapes and orientation in relation to the axes is a subject which appears to be beyond the possibilities of mathematical solution from our present knowledge, and much of the data will have to be acquired by experimental methods. It is for this reason imperative that, in any experimental work on this subject broadcast in papers, the writer should define the quality of the quartz and the accuracy of working as far as circumstances permit. In this connexion it is very gratifying to find that some of Dye's very beautiful photographs of the vibrations of quartz plates, taken by stroboscopic means, are included, but it is a pity that the subject is dismissed in a few lines.

The number of illustrations is in keeping with the size of the book, there being in all 125 referred to in the text.

As might be expected, the work done at the National Physical Laboratory on piezo-electric resonators finds a place in the chapter under this heading, but it by no means forms the major portion of the matter in this chapter; all outstanding papers up to the date of compilation being either commented upon or extracted.

The arrangement of the book in sections, whilst doubtless possessing advantages from the compiler's point of view, has the disadvantage that the sections cannot be located so quickly as the numbered pages. It is to be hoped that in the next issue the general index will be extended and the page numbers quoted in place of the section numbers.

S. J. U.

Die optische Werkstatt: Handbuch der Arbeitsverfahren und Prüfmethoden für die Fertigung von Optik. Von Dr WALDEMAR EWALD, unter Mitarbeit von Dr HANS SCHULZ und Dr FRANZ WEIDERT. Pp. viii + 227; 169 Figs. and 3 Interpolation Tables. (Berlin: Gebrüder Borntraeger, 1930.) Price RM. 20.

This is the first effort which has come to my knowledge of putting into collected form the various methods which have been brought into use for the mechanisation of the glass-grinding industry as applied to optical instruments. Modern machine tool building has

made tremendous strides to serve the metal-working industry, and there exist any number of publications dealing with the latest developments; not so in the optical glass-working industry. The book should be welcomed on this score alone.

The first part of the book deals with the usual raw materials as used by opticians, namely, glass, crystals, etc.

The second chapter forms the main part of the book; it is headed "The Workshop" and is divided into four parts A, B, C, and D. A contains a few pages of matter in connexion with general arrangements, cements, testing accuracy, and silvering. B deals in fair length with the machinery devised for the rapid production of ordinary lenses and spectacle lenses of the more complex forms. C contains a fairly comprehensive list of testing instruments, but it is strange that no mention is made of the Hilger interferometer in its various forms for testing optical elements.

The book finishes with a set of interpolation tables to facilitate the calculation of the refractive indices for C , D , F , and G' from the indices measured for a set of lines selected from the spectrum of the mercury vapour arc.

The subject-matter of the book has been well selected, and is clearly stated, while the illustrations are exceedingly well presented. The book should be well received by all practical people connected with the optical industry.

J. W. H.

I.G. Farbenindustrie Aktiengesellschaft: Veröffentlichungen des wissenschaftlichen Zentral-Laboratoriums der photographischen Abteilung (Agfa). Band 1. Pp. 155; 94 Figs. and 3 Plates. (Leipzig: S. Hirzel, 1930.) Price RM. 10.

This volume is a collection of technical papers dealing with work carried out in the photographic section of the I.G. Farbenindustrie Aktiengesellschaft. Some of them have already been published in photographic journals, while others are published here for the first time. The first paper is one by J. Eggert on the present position of the silver nucleus theory of the latent photographic image. The three-dimensional surface which represents the blackening of a photographic layer as a function of the light intensity and the time is discussed by H. Arens. A paper by H. Arens and J. Eggert deals with the principles of colour sensitometry with the aid of the spectrum and colour tables, and A. Hübl's system of colour sensitometry is criticised by M. Biltz and E. Heisenberg. A second group of papers deals with problems of applied photography. W. Dieterle describes some of the substances used for sensitising plates in the infra-red region and discusses some of the applications of infra-red photography. J. Eggert and R. Schmidt contribute a paper dealing with problems connected with the production of sound films. The causes of the mottling effect sometimes obtained when making half-tone copies are discussed by K. Hebbel and a method of overcoming this defect is described. A paper by W. Barth deals with the production and observation of stereo-X-ray photographs, and an instrument for measuring X-ray doses is described by F. Luft. The volume concludes with two papers, one by E. Heisenberg on the thermal properties of cellulose and its derivatives, and the other by M. Hagedorn and P. Moeller on the dependence of some physical properties of cellulose derivatives on chemical constitution. This collection of papers is sure to be of value to all who are interested in the scientific aspects of photography.

J. S. A.